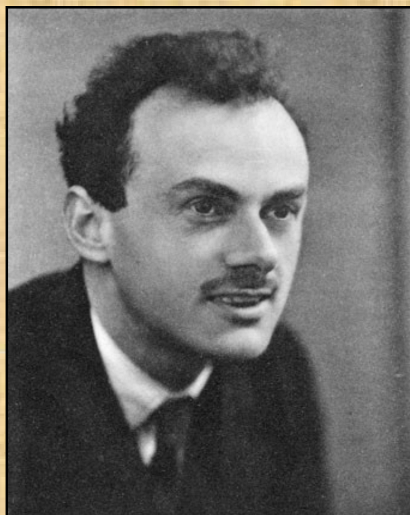
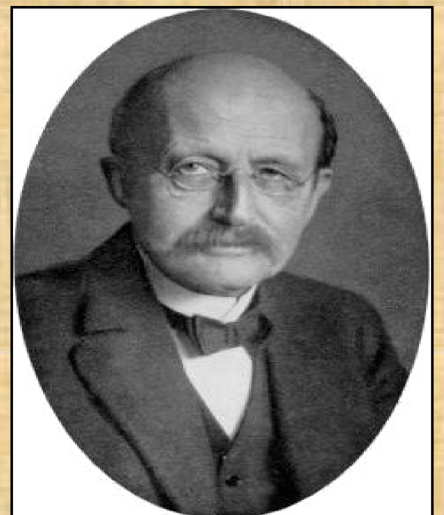
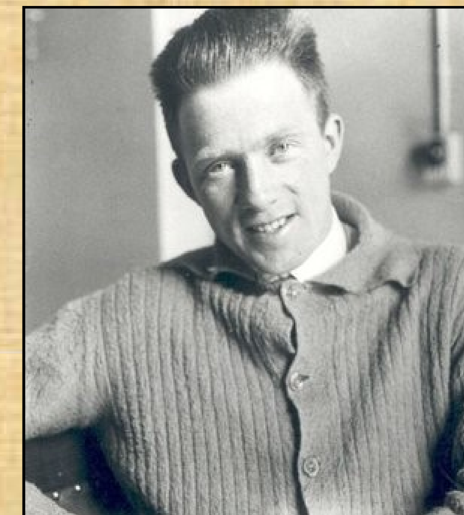
Quantization, Wave Function & Operators (Lesson 7)

Compiled by

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Topics of Interest:

- Quantisation, Wave Particle Duality
- Wave functions and operators
- Measurable physical quantities and associated operators – Correspondence principle
- The Schrödinger Equation
- The Uncertainty principle

You already know h is the Planck constant in J.s

$$p = \hbar\ k = \frac{h}{\lambda} \quad ; \quad E = \hbar\ \omega = h\nu = \frac{h}{T} \quad \text{et} \quad v_\varphi = \frac{E}{p}$$

Louis de BROGLIE
French
(1892-1987)

Max Planck (1901)
Göttingen

From Black-Body Radiation

Why a decrease for small λ ? Quantification!

Max Planck
(1901)
Göttingen

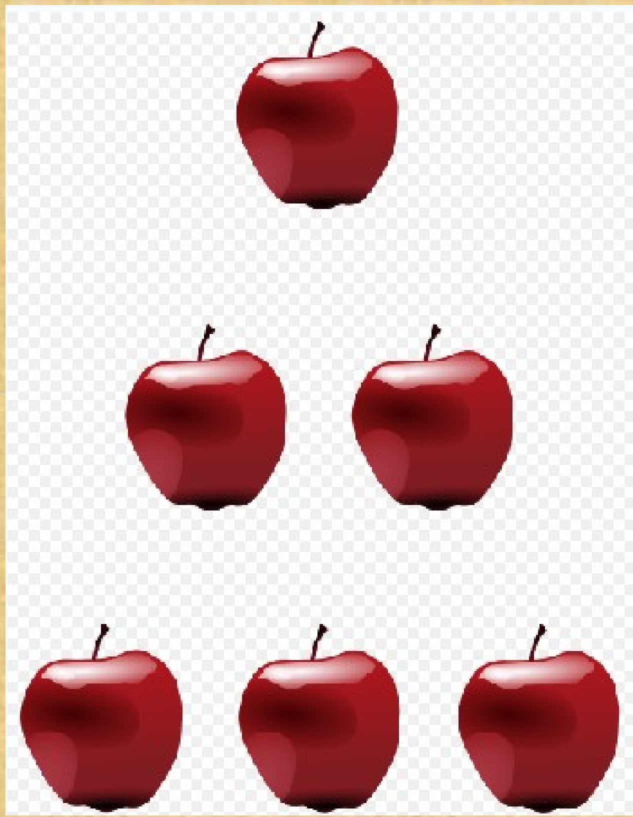
Numbering rungs of ladder introduces quantum numbers (here equally spaced)

Quantum Numbers

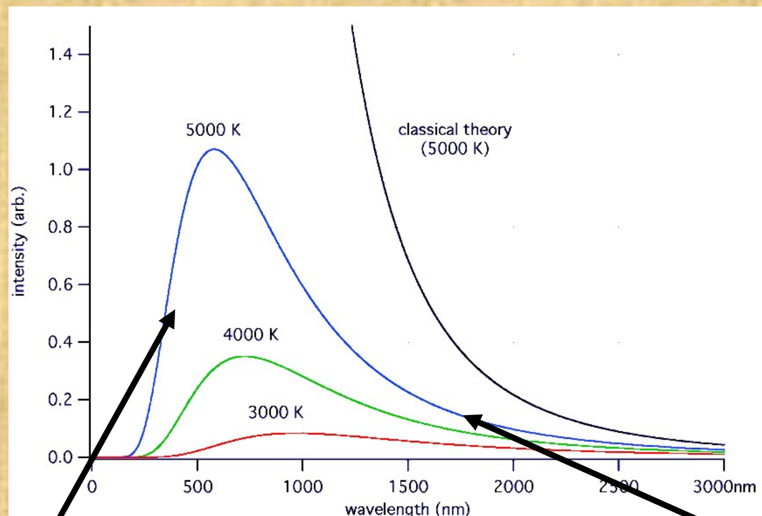
✓In mathematics, a natural number (also called counting number) has two main purposes:

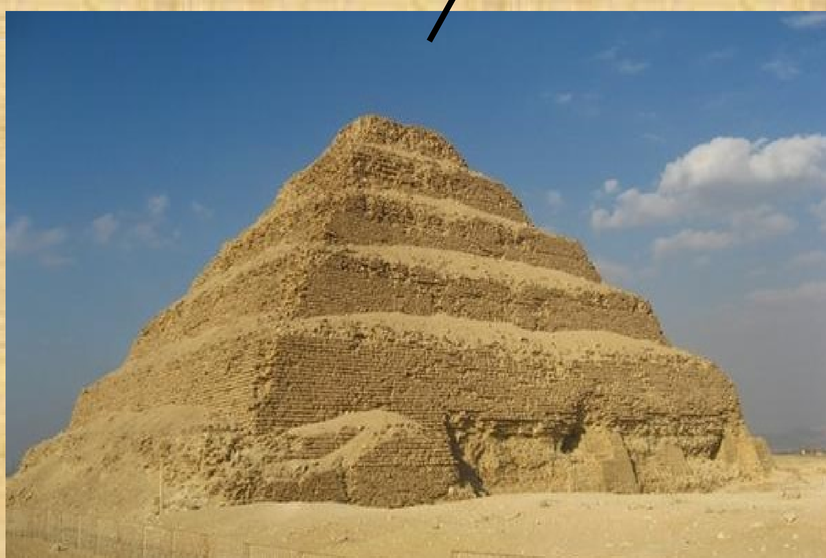
✓they can be used for counting ("there are 6 apples on the table"), and also

✓they can be used for ordering ("this is the 3rd largest city in the country").

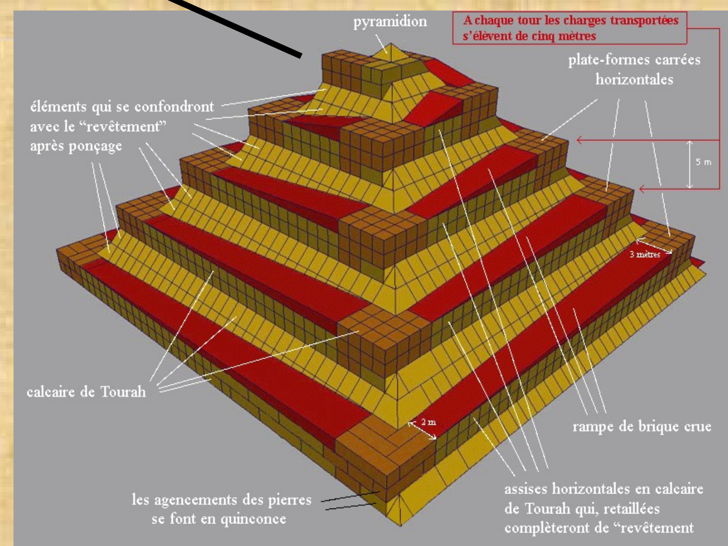


Black-Body Radiation, Quantification





Steps too hard to climb
(Pyramid nowadays)

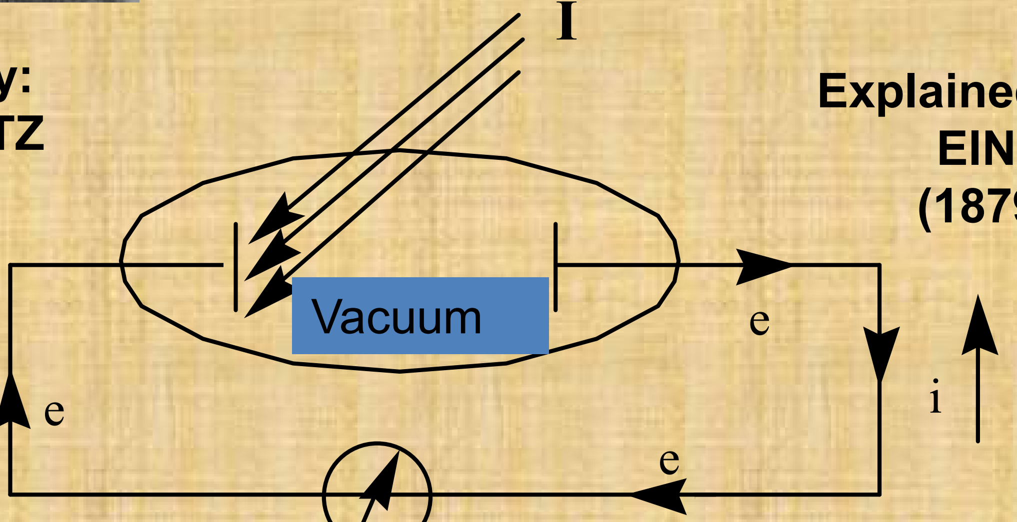


Easy slope, ramp
(Pyramid under construction)

Photoelectric Effect (1887-1905)

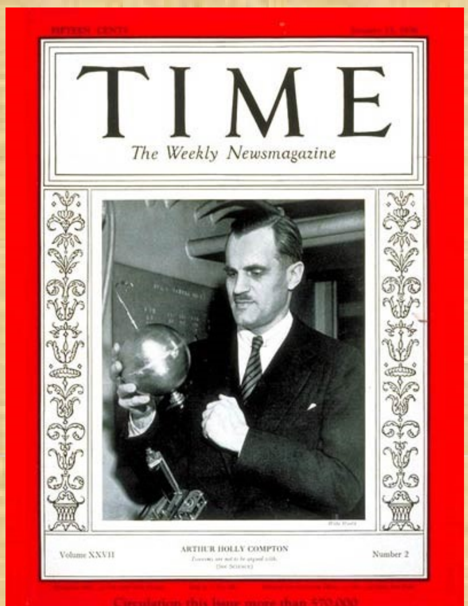
Discovered by: Heinrich HERTZ (1857-1894)

Explained by: Albert EINSTEIN (1879-1955)

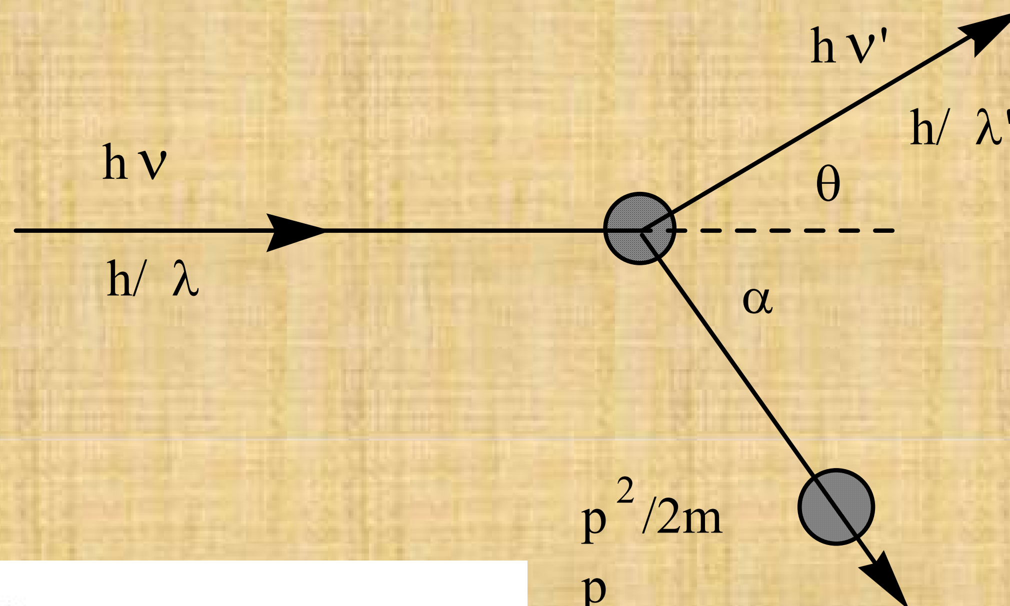


Compton Effect 1923

playing billiards assuming l=h/p



Arthur Holly Compton
American
1892-1962



Energy Conservation

$$h\,\frac{c}{\lambda} = h\frac{c}{\lambda'} + \frac{p^2}{2m} \qquad (\nu = \frac{c}{\lambda})$$

Momentum Conservation (projection on x)

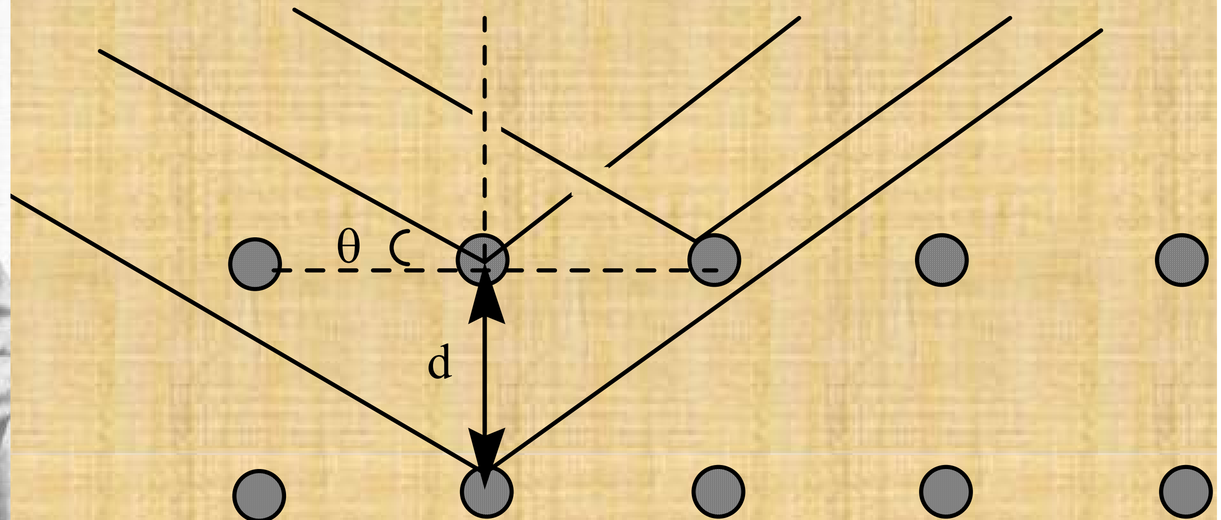
$$\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\theta + p\cos\alpha$$

Momentum Conservation (projection on y)

$$0 = \frac{h}{\lambda'}\sin\theta - p\sin\alpha$$

Davisson and Germer 1925

Clinton Davisson
Lester Germer
In 1927



$$2d \sin\theta = k\lambda$$

Diffraction is similarly observed using a mono-energetic electron beam

Bragg law is verified assuming l=h/p

Wave–Particle Duality

In physics and chemistry, **wave–particle duality** is the concept that all matter and energy exhibits both wave-like and particle-like properties. A central concept of quantum mechanics, duality, addresses the inadequacy of classical concepts like "particle" and "wave" in fully describing the behavior of small-scale objects. Various interpretations of quantum mechanics attempt to explain this apparent paradox.

In Macroscopic World:

- A basket of cherries
- Many of them (identical)
- We can see them and taste others
- Taking one has negligible effect
- Cherries are both red and good

In Microscopic World:

- A single cherry
- Either we look at it without eating; It is red or, we eat it, it is good. (mutually exclusive)
- You can not try both at the same time. The cherry could not be good and red at the same time. (uncertainty & duality)

Slot machine "one-arm bandit"

After introducing a coin, you have 0 coin or X coins.

A measure of the profit has been made: profit = X

de Broglie Relation from Relativity

Popular expressions of relativity: m_0 is the mass at rest, m in motion

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad \text{and} \qquad E = m\,c^2$$

E like to express E(m) as E(p) with p=mv

$$E^2 = m^2 c^4 \left(1 - \frac{v^2}{c^2}\right) + p^2c^2 = m_0^2 c^4 + p^2c^2$$

$$= \sqrt{m_0^2 c^4 + p^2c^2} = m_0 c^2 \left(1 + \frac{p^2}{m_0^2c^2}\right)^{1/2} = E_i + \frac{p^2}{2m_0} + \dots$$

E_i + T + $E_{relativistic}$ + ….

de Broglie Relation from Relativity

$$E^2 = m_0^2 c^4 + p^2c^2$$

Application to a photon (m_0=0)

$$E = h\nu$$

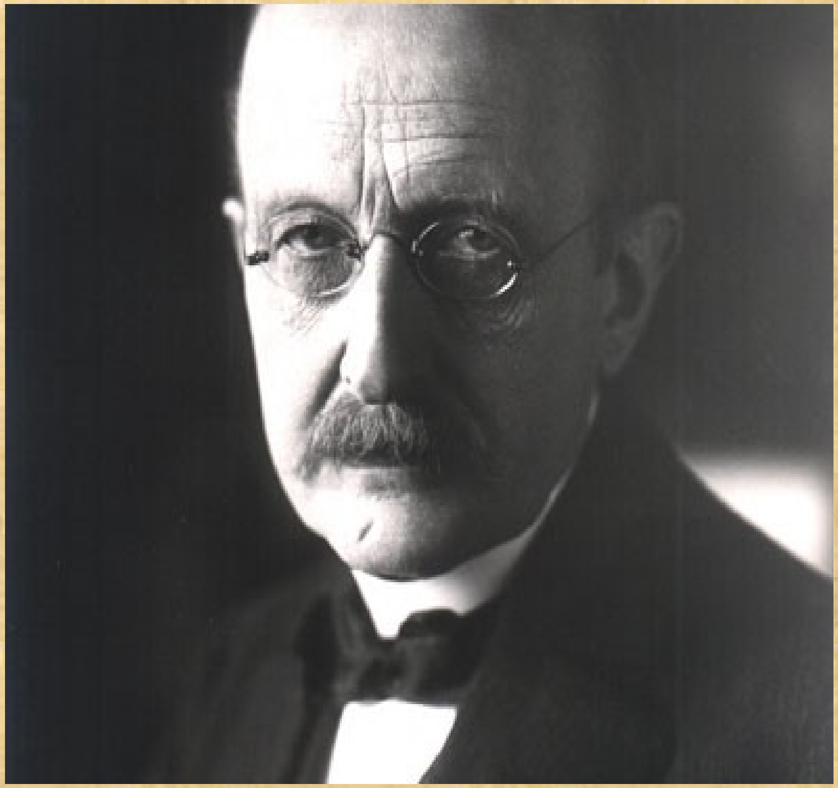
To remember

$$E = pc \rightarrow pc = h\nu$$

$$p = \frac{h\nu}{c} = \frac{h}{\lambda} = \frac{E}{c}$$

$$\lambda = \frac{h}{p}$$

To remember



Max Planck

Useful to remember to relate energy and wavelength

$$\lambda = \frac{hc}{E} \rightarrow \lambda\,(\text{Å}) = \frac{12410}{E(\text{eV})}$$

A New Mathematical Tool: Wave Functions & Operators

✓Each particle may be described by a wave function **Y**(x,y,z,t), real or complex, having a single value when position (x,y,z) and time (t) are defined.

✓If it is not time-dependent, it is called stationary.

✓The expression **Y**$=Ae^{i(pr-Et)}$ does not represent one molecule but a flow of particles: a plane wave

Wave Functions Describing One Particle

To represent a single particle Ψ(x,y,z) that does not evolve in time, Ψ(x,y,z) must be finite (0 at ∞).

In QM, a particle is not localized but has a probability to be in a given volume:

dP= $\Psi^* \Psi$ dV is the probability of finding the particle in the volume dV. Around one point in space, the density of probability is dP/dV= $\Psi^* \Psi$

Ψ has the dimension of $L^{-1/3}$

Integration in the whole space should give one. That means Ψ is normalized.

$$\int_{total\_space} \Psi^* \Psi dV = 1$$

Operators Associated to Physical Quantities

We cannot use functions (otherwise we would end with classical mechanics)

Any physical quantity is associated with an operator.

An operator O is "the recipe to transform Y into Y' "

We write: O Y = Y'

If O Y = o Y (o is a number, meaning that O does not modify Y, just a scaling factor), we say that Y is an eigen function of O and o is the eigen value.

We have solved the wave equation O Y = o Y by finding simultaneously Y and o that satisfy the equation.

o is the measure of O for the particle in the state described by Y.

O is a Vending machine (cans)

Introducing a coin, you get one can.

No measure of the gain is made unless you sell the can (return to coins)

Slot machine (one-arm bandit)

Introducing a coin, you have 0 coin or X coins.

A measure of the profit has been made: profit = X

Examples of Operators in Mathematics : Say P parity

$$Pf(x) = f(-x)$$

Even function : no change after $x \rightarrow -x$

Odd function : f changes sign after $x \rightarrow -x$

$y=x^2$ is even

$y=x^3$ is odd

$y= x^2 + x^3$ has no parity: $P(x^2 + x^3) = x^2 - x^3$

Examples of Operators in Mathematics: A

$$A = \frac{d^2}{dx^2} - x^2$$

Apply to the wave function $y = e^{-x^2/2}$

$$A = \frac{d^2y}{dx^2} - x^2y = \frac{d^2e^{-x^2/2}}{dx^2} + x^2\,e^{-x^2/2}$$

$$A\,y = \frac{d(-x\,e^{-x^2/2}\,)}{dx} - x^2\,e^{-x^2/2} = -\;e^{-x^2/2} + x^2\,e^{-x^2/2} - x^2\,e^{-x^2/2}$$

$$A\,y = -\;e^{-x^2/2} = -y$$

Here, y is an eigen function; the eigen value is -1

Linearity

The operator O is said to be linear:

If $O\,(aY_1 + bY_1) = O\,(aY_1\,) + O(\,bY_1)$

Normalization

An eigen function remains an eigen function when multiplied by a constant.

That means, O(IY)= o(IY) thus it is always possible to normalize a finite function

$$\int_{total\_space} \Psi^* \Psi dV = N \quad \text{taking } \Psi' = \frac{1}{\sqrt{N}} \Psi \quad \text{gives} \quad \int_{total\_space} \Psi'^* \Psi' dV = 1$$

Dirac notation is like: $<\Psi|\Psi>$

Mean Value

•If Y_1 and Y_2 are associated with the same eigen value o: then,

$O(aY_1 + bY_2) = o(aY_1 + bY_2)$

•If not, then

•$O(aY_1 + bY_2) = o_1(aY_1) + o_2(bY_2)$

we define $\bar{o} = (a^2 o_1 + b^2 o_2)/(a^2 + b^2)$

$$\Psi^* \, O \, \Psi = \Psi^* \, o \, \Psi$$

o is a multiplying factor

$$\int_\infty (\Psi^* O \Psi) dV = o \int_\infty (\Psi^* \Psi) dV$$

Dirac notations

$$o = \frac{\int_\infty (\Psi^* O \Psi) dV}{\int_\infty (\Psi^* \Psi) dV} = \frac{<\Psi | O | \Psi>}{<\Psi | \Psi>}$$

Sum, Product and Commutation of Operators

$(A+B)\Psi = A\Psi + B\Psi$

$(AB)\Psi = A(B\Psi)$

Eigen values

Wave functions

operators →

	$y_1=e^{4x}$	$y_2=x^2$	$y_3=1/x$
d/dx	4	--	--
x3	3	3	3
x d/dx	--	2	-1

Sum, Product and Commutation of Operators

[A,C] = AC-CA ≠ 0

[A,B] = AB-BA = 0

[B,C] = BC-CB = 0

$$[A,C]\,(y) = AC(y) - CA(y) = \frac{d}{dx}\left[x\,\frac{dy}{dx}\right] - x\left[\frac{d}{dx}\left(\frac{dy}{dx}\right)\right]$$

$$[A,C]\,(y) = x\,\frac{d^2y}{dx^2} + \frac{dy}{dx} - x\,\frac{d^2y}{dx^2} = A(y)$$

$$[A,C] = A$$

	$y_1=e^{4x}$	$y_2=x^2$	$y_3=1/x$
A = d/dx	4	--	--
B = x3	3	3	3
C= x d/dx	--	2	-1

non compatible operators

Compatibility, Incompatibility of Operators

$[A,C]=AC-CA\neq 0$

When the operators commute, the physical quantities may be simultaneously defined (compatibility)

$[A,B]=AB-BA=0$

$[B,C]=BC-CB=0$

When operators do not commute, the physical quantities can not be simultaneously defined (incompatibility). That means they are guided by the Heisenberg's Uncertainty Principle.

compatible operators

non compatible operators

	$y_1=e^{4x}$	$y_2=x^2$	$y_3=1/x$
$A = d/dx$	4	--	--
$B = {}_{x}3$	3	3	3
$C= x\, d/dx$	--	2	-1

Look! x and d/dx do not commute, they are incompatible

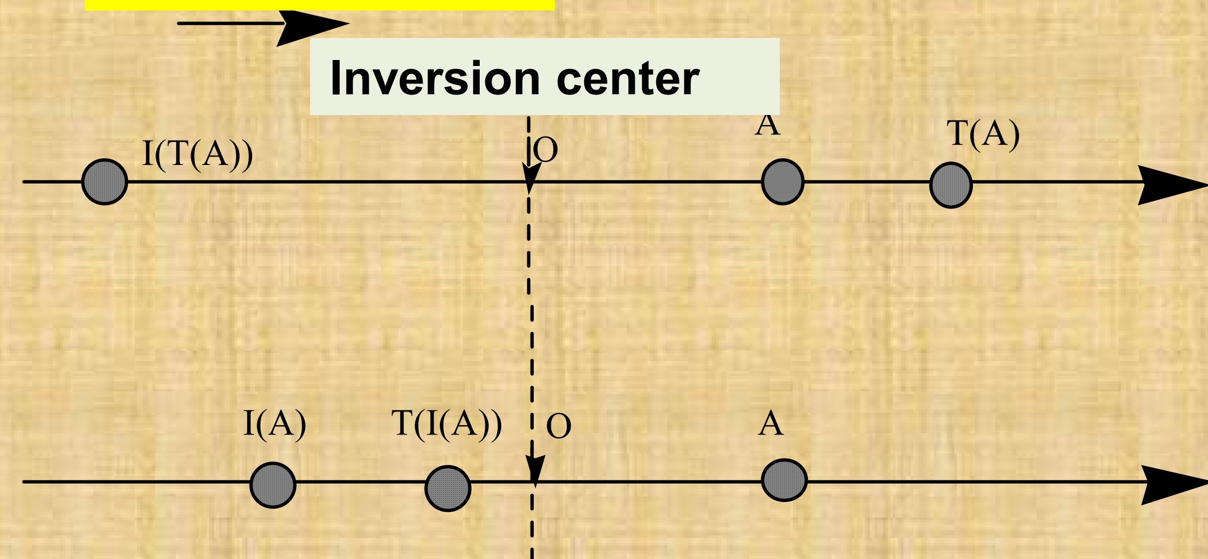
$$[d/dx, x] = d/dx(x\ \ ) - x(d/dx)$$

$$[d/dx, x](y) = \frac{d(xy)}{dx} - x\frac{dy}{dx} = x\frac{dy}{dx} + y - x\frac{dy}{dx} = y$$

$$[d/dx, x] = +1$$

Look! Translation and inversion also do not commute, they are incompatible too……….

Translation vector



Introducing New Variables!

Now it is time to give a physical meaning to the wave function. Where, p is the momentum, E is the Energy, $h=6.62\ 10^{-34}$ J.s

$$p = \frac{h}{\lambda} \text{ and } E = h\nu$$

$$\hbar = \frac{h}{2\pi}$$

$$\Psi = A\, e^{i(kr-\omega t)} = A\, e^{2\pi i(\frac{r}{\lambda} - \nu t)} = A\, e^{\frac{i}{\hbar}(pr-Et)}$$

General Expression for Plane Waves

$$\Psi = A\, e^{i(kr-\omega t)} = A\, e^{2\pi i(\frac{r}{\lambda} - \nu t)} = A\, e^{\frac{i}{\hbar}(pr-Et)}$$

This represents a (monochromatic) beam, a continuous flow of particles with the same velocity (mono kinetic).

k, l, w, n, p and E are perfectly defined

R (position) and t (time) are not defined.

YY*=A^2=constant everywhere; there is no localization.

If E = constant, this is a stationary state, independent of t (time) which is not defined.

Correspondence Principle 1913/1920

Niels Henrik David Bohr
Danish
1885-1962

For every physical quantity one can define an operator. The definition uses formulae from classical physics replacing quantities involved by the corresponding operators.

QM is then built from classical physics in spite of demonstrating its limits.

Operators p and H

We would use expression of the plane wave stated earlier as it allows defining exactly both p and E.

Momentum and Energy Operators

$$\Psi(r,t)=\Psi(r)\,\Psi(t) = A\, e^{\frac{i}{\hbar}(pr)}\; e^{\frac{i}{\hbar}(-Et)} .$$

$$\Psi(r) = Ae^{\frac{i}{\hbar}(pr)} \qquad \Psi(t) = e^{\frac{i}{\hbar}(-Et)}$$

$$\frac{\partial \Psi}{\partial r} = \frac{ip}{\hbar}\Psi \quad \text{and} \quad \frac{\partial \Psi}{\partial t} = -\frac{iE}{\hbar}\Psi$$

$$\text{therefore } \mathbf{p} = \frac{\hbar}{i}\frac{\partial}{\partial r} \quad \text{and} \quad \mathbf{E} = i\hbar\frac{\partial}{\partial t}$$

Remember this during this chapter

Stationary State E = constant

$$\Psi(t) = e^{\frac{i}{\hbar}(-Et)}.$$

$$i\hbar \frac{\partial \Psi(r,t)}{\partial t} = E\Psi(r,t) = E\Psi(r)\Psi(t)$$

Remember this for 3 slides after

Kinetic Energy Operator

Classical $T = \frac{p^2}{2m}$ **quantum operator** $P^2 = \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2}$

In 3D : $T = \frac{-\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) = \frac{-\hbar^2}{2m} \Delta$

Calling $\Delta = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)$ **the Laplacian**

Pierre Simon, Marquis de Laplace
(1749 -1827)

Correspondence Principle: Angular Momentum

Classical expression

$l_Z = xp_y - yp_x$

Quantum expression

$$L_Z = -i\hbar \left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$$

Quantum-mechanical Operators

Operator name	Symbol	Form	Notes
position (in x direction, for example)	$\hat{x}$	x	There are also corresponding operators for y, z.
position (three-dimensional, Cartesian coordinates)	$\hat{\mathbf{r}}$	$(x,y,z) = x\vec{x} + y\vec{y} + z\vec{z}$	
dipole moment (three-dimensional, Cartesian coordinates)	$\hat{\boldsymbol{\mu}}$	$\sum_{j=\text{all atoms}} q_j \mathbf{r}_j$	q is the charge on each atom; **r** is the position of each atom.
dipole moment (one atom in spherical coordinates)	$\hat{\boldsymbol{\mu}}$	$-er\left(\vec{\mathbf{x}}\sin\theta\cos\varphi + \vec{\mathbf{y}}\sin\theta\sin\varphi + \vec{\mathbf{z}}\cos\theta\right)$	
del	∇	$\vec{\mathbf{x}}\frac{\partial}{\partial x} + \vec{\mathbf{y}}\frac{\partial}{\partial y} + \vec{\mathbf{z}}\frac{\partial}{\partial z}$	
Laplacian (three dimensional, Cartesian coordinates)	∇^2	$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$	
Laplacian (spherical coordinates)	∇^2	$\frac{1}{r^2}\left(\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right)$	
linear momentum (in x direction, for example)	$\hat{p}_x$	$-i\hbar\frac{\partial}{\partial x}$	There are also corresponding operators for y, z.

linear momentum (3 dimensions)	$\hat{\mathbf{p}}$	$-i\hbar\nabla$	
z-component of angular momentum	$\hat{M}_z$	$-i\hbar\frac{\partial}{\partial\varphi}$	
the square of total angular momentum	$\hat{M}^2$	$-\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right]$	
electronic angular momentum	$\hat{\mathbf{L}}$	$\vec{\mathbf{x}}\,\hat{L}_x + \vec{\mathbf{y}}\,\hat{L}_y + \vec{\mathbf{z}}\,\hat{L}_z$	Operator has the same form as the angular momentum operator, $\hat{M}$
total energy (Hamiltonian)	$\hat{H}$	$\hat{T} + \hat{V}$	The potential energy (V) depends on the specific system being modeled. Additional energy terms can be added (e.g., interactions with electric or magnetic fields.)
kinetic energy for a single particle (Cartesian coordinates)	$\hat{T}_x$	$\left(\frac{-\hbar^2}{2m}\right)\frac{\partial^2}{\partial x^2}$	one-dimensional expression
	$\hat{T}$	$\left(\frac{-\hbar^2}{2m}\right)\nabla^2$	generalized expression
kinetic energy for a single particle (spherical coordinates)	$\hat{T}$	$\frac{-\hbar^2}{2\mu r^2}\left[\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right]$	

potential energy for a particle in a box	$\hat{V}$	$V = 0$ inside box $(0 \le x \le L)$ $V = \infty$ outside box $(x < 0, x > L)$	The box can be extended to 2 or 3 dimensions and the length of the box, L, can be different in each direction.
harmonic oscillator potential	$\hat{V}$	$\frac{K}{2}\left(x - x^{e}\right)^{2}$	For diatomic molecules; K is the force constant; x^e is the equilibrium bond length.
		$\frac{K}{2}\hat{Q}^{2}$	For polyatomic molecules; $\hat{Q}$ is the operator for the magnitude of the normal coordinate.
Morse (anharmonic) potential	$\hat{V}$	$D_{e}\left(1 - e^{-\beta x}\right)^{2}$	where x is the displacement of the oscillator from its equilibrium position, and D_e and β are constants or parameters.
Coulombic potential energy	$\hat{V}$	$\frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$	q is the charge on each particle; r is the distance between particles.
electric field	$\hat{\mathbf{E}}$	$\mathbf{E}$	Simple multiplicative operator as long as the field is constant over the area of interaction with the system.
magnetic field	$\hat{\mathbf{B}}$	B	Operator is multiplicative and can be written without the caret.

magnetic moment of an electron	$\hat{\boldsymbol{\mu}}_m$	$-\frac{e}{2m_e}\hat{\mathbf{L}}$	
energy of interaction with a magnetic field	$\hat{H}_m$	$-\hat{\boldsymbol{\mu}}_m \bullet \mathbf{B}$	general form
		$\frac{e}{2m_e}\hat{\mathbf{L}} \bullet \mathbf{B}$	the particle is an electron and the magnetic field is directed along the z-axis
		$\frac{e B_z}{2 m_e}\hat{L}_z$	
spin orbit interaction	$\hat{H}_{s-o}$	$\lambda\, \hat{\mathbf{S}} \bullet \hat{\mathbf{I}}$	
Coulomb operator	$\hat{J}$	$\hat{J}_j(1)\, \varphi_i(1) = \left[\int \varphi_i^*(2)\, \frac{1}{r_{12}}\, \varphi_i(2)\, d\tau_2\right] \varphi_i(1)$	
Exchange operator	$\hat{K}$	$\hat{K}_j(1)\, \varphi_i(1) = \left[\int \varphi_j^*(2)\, \frac{1}{r_{12}}\, \varphi_i(2)\, d\tau_2\right] \varphi_j(1)$	
Fock operator	$\hat{F}$	$\hat{H}^0 + \sum_{j=1}^{N}(2\hat{J}_j - \hat{K}_j)$	
		$-\frac{\hbar^2}{2m}\nabla^2 - \frac{Z e^2}{4\pi\varepsilon_0 r} + \sum_{j=1}^{N}(2\hat{J}_j - \hat{K}_j)$	

Time-Dependent Schrödinger Equation

Erwin Rudolf Josef Alexander
Schrödinger
Austrian
1887 –1961

Without potential E = T
With potential E = T + V

$$i\hbar\frac{\partial\Psi(r,t)}{\partial t} = -\frac{\hbar^2}{2m}\Delta\Psi(r,t)$$

$$i\hbar\frac{\partial\Psi(r,t)}{\partial t} = -\frac{\hbar^2}{2m}\Delta\Psi(r,t) + V(r,t)\Psi(r,t)$$

Schrödinger Equation for stationary states

$$i\hbar \frac{\partial \Psi(r,t)}{\partial t} = -\frac{\hbar^2}{2m} \Delta\Psi(r,t) + V(r,t)\Psi(r,t)$$

$$\Psi(r,t) = \Psi(r).\Psi(t) = A \exp\left(\frac{-iEt}{\hbar}\right) \Psi(r)$$

$$i\hbar \frac{\partial \Psi(r).\Psi(t)}{\partial t} = -\frac{\hbar^2}{2m} \Delta\Psi(r).\Psi(t) + V(r)\Psi(r).\Psi(t) = E\,\Psi(r).\Psi(t)$$

$$-\frac{\hbar^2}{2m} \Delta\Psi(r) + V(r)\Psi(r) = E\,\Psi(r)$$

Potential energy

Kinetic energy

Total energy

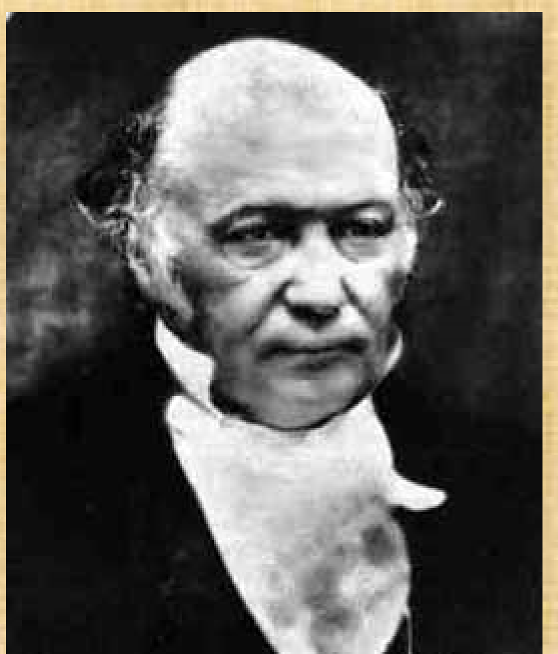
Schrödinger Equation for Stationary States

$$-\frac{\hbar^2}{2m}\Delta\Psi(r) + V(r)\Psi(r) = E\,\Psi(r)$$

Remember

$$\mathbb{H}\,\Psi(x,y,z) = E\Psi(x,y,z) \quad \text{with} \quad \mathbb{H} = -\frac{\hbar^2}{2m}\Delta + \mathbb{V}$$

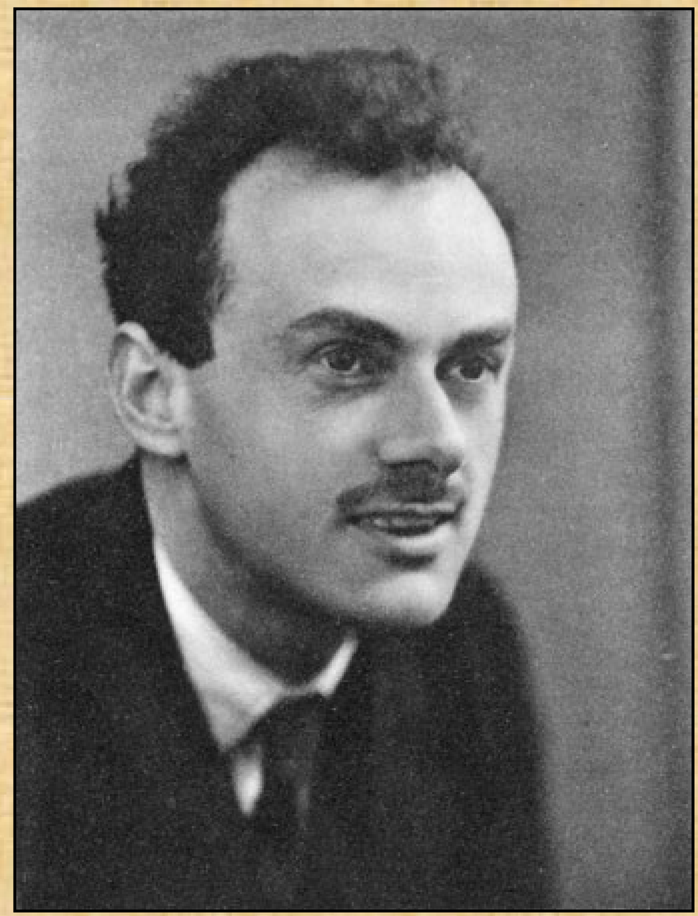
H is the hamiltonian



Sir William Rowan Hamilton
Irish 1805-1865

Half penny bridge in Dublin

Chemistry is Nothing but an Application of Schrödinger Equation (Dirac)

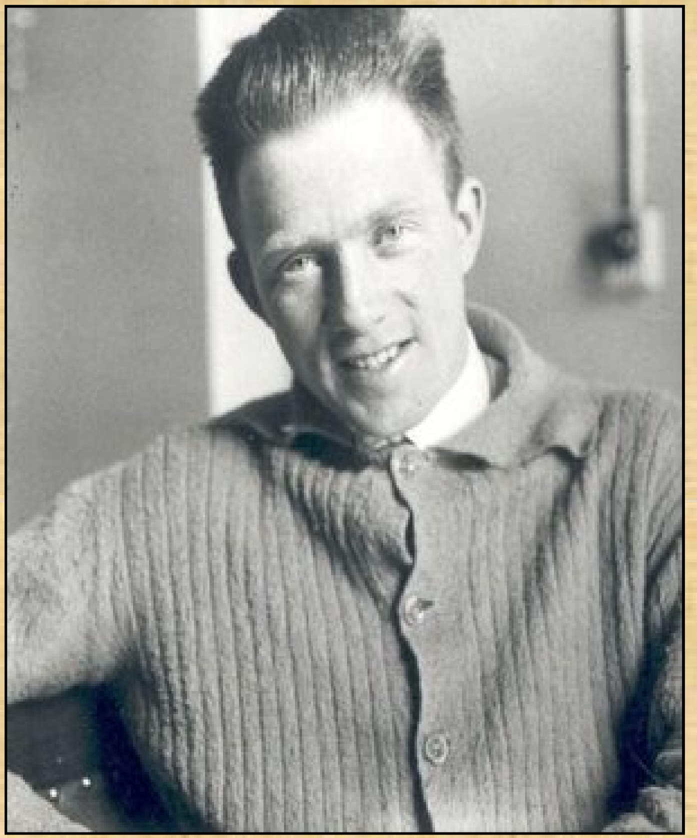


Paul Adrien Dirac
1902 – 1984

$< \Psi | \Psi >$ $< \Psi | O | \Psi >$

Remember this Dirac notation, we would need later

Uncertainty Principle



Werner Heisenberg
German
1901-1976

The **Heisenberg uncertainty principle** states that locating a particle in a small region of space makes the momentum of the particle uncertain; and conversely, that measuring the momentum of a particle precisely makes the position uncertain. Remember, we already have seen incompatible operators!

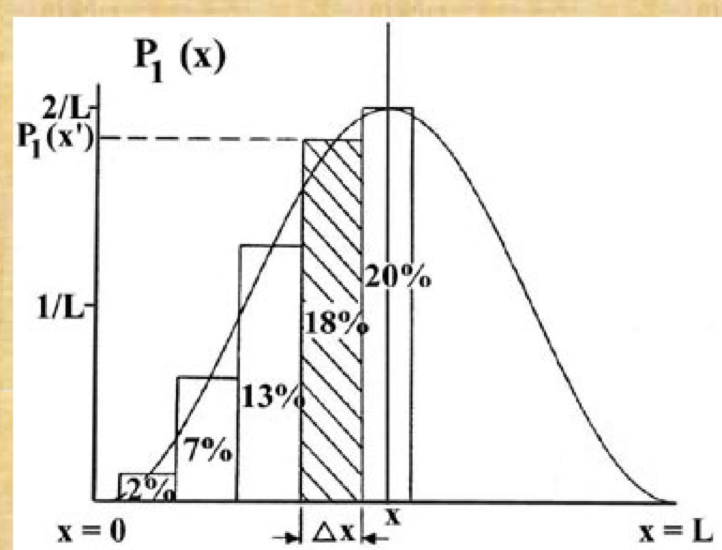
▪It is not surprising to find that quantum mechanics does not predict the position of an electron exactly. Rather, it provides only a ***probability*** as to where the electron will be found.

▪We shall illustrate the probability aspect in terms of the system of an electron confined to motion along a line of length *L*. Quantum mechanical probabilities are expressed in terms of a distribution function.

▪For a plane wave, p is defined and the position is not.

▪With a superposition of plane waves, we introduce an uncertainty on p and we localize. Since, the sum of 2 wave fucntions is neither an eigen function for p nor x, we have average values.

▪With a Gaussian function, the localization below is 1/2p



p and x do not commute and are hence incompatible.

For a plane wave, actually p is known and x is not, as ($Y^*Y=A^2$ everywhere.

Let's superpose two waves…

$$\lambda_1 = \lambda + \frac{\Delta\lambda}{2} \text{ et } \lambda_2 = \lambda - \frac{\Delta\lambda}{2} .$$

this introduces a delocalization for p and may be localized x.

At the origin x=0 and at t=0, now if we want to increase the total amplitude, the two waves Y_1 and Y_2 are taken in phase. At ± Dx/2 we want to impose them out of phase. The position is therefore known for x ± Dx/2; the waves will have wavelengths.

$$\frac{x}{\lambda_1} - \frac{x}{\lambda_2} = \frac{1}{2} \quad \rightarrow \quad \frac{\frac{\Delta x}{2}}{\lambda + \frac{\Delta\lambda}{2}} - \frac{\frac{\Delta x}{2}}{\lambda - \frac{\Delta\lambda}{2}} = \frac{1}{2}$$

$$\frac{\frac{\Delta x}{2}(\lambda - \frac{\Delta\lambda}{2}) - \frac{\Delta x}{2}(\lambda + \frac{\Delta\lambda}{2})}{(\lambda + \frac{\Delta\lambda}{2})(\lambda - \frac{\Delta\lambda}{2})} = \frac{1}{2} = \frac{-\Delta x \Delta\lambda}{2\lambda^2}$$

$$\Delta x \Delta\lambda = -\lambda^2 \qquad p = \frac{h}{\lambda} \rightarrow dp = \frac{-h\, d\lambda}{\lambda^2}$$

making $\Delta p = dp$ and $\Delta\lambda = d\lambda$

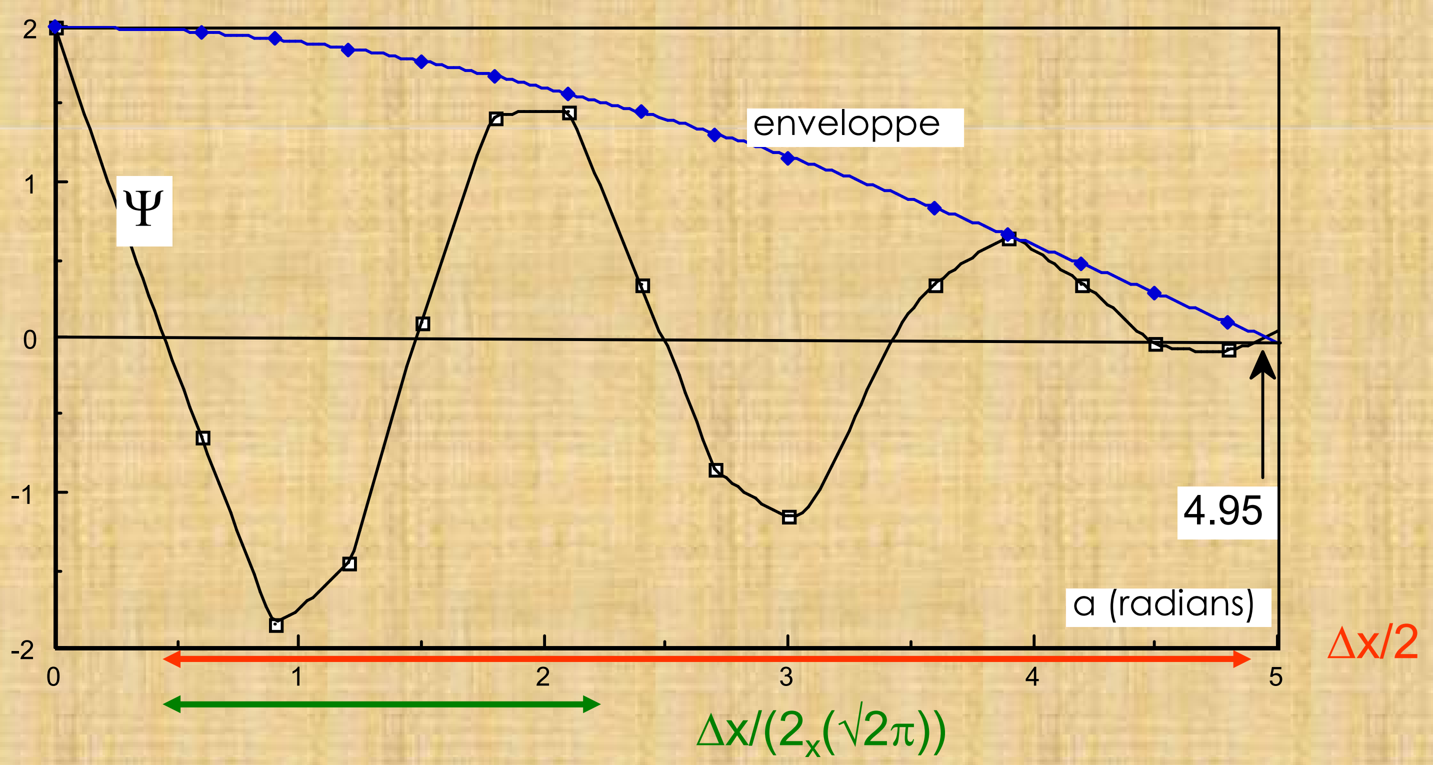
$$\rightarrow \quad \boxed{\Delta x . \Delta p = h}$$

Superposition of Two Waves

$$\Psi = \cos(2\pi(\frac{x}{0.9} - \nu t)) + \cos(2\pi(\frac{x}{1.1} - \nu t)) = 2\cos(\frac{\pi x}{9.9})\cos(2\pi(\frac{x}{0.99} - \nu t))$$

$$\frac{\Delta x}{2} = 4.95 \qquad \Delta\lambda = 1.1\text{-}0.9 = 0.2$$

$$\Delta x . \Delta\lambda = 0.99 \text{ close to } \lambda^2 = 1$$



Uncertainty Principle

Werner Heisenberg
German
1901-1976

A more accurate calculation localizes the particles more and therefore one gets;

$$\Delta x . \Delta p = \hbar \qquad \Delta E . \Delta t = \hbar$$

$$\hbar = 1.0536\ 10^{-34}\ \text{J.s}$$

$$\Psi = A \exp\left(\frac{2\pi i}{h}(px - Et)\right)$$

x and p or E and t play symmetric roles in the plane wave expression;

Therefore, there are the two main UNCERTAINTY PRINCIPLES

Remember:

This much about Functions and Operators for now. More on them in the coming lessons!

THANK YOU!

ANY QUESTION?