Wave Nature of Matter: Made Easy (Lesson 3)

Matter behaving as a wave? Ridiculous!

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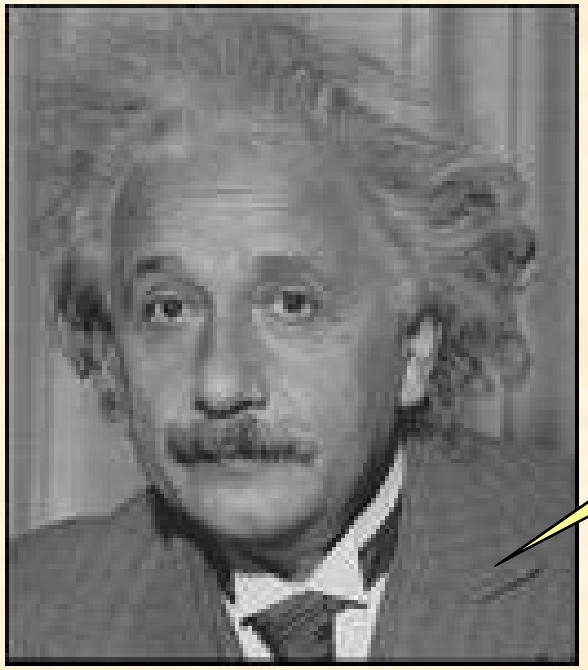
Remember?

I showed you earlier how Einstein (in 1905) showed that the photoelectric effect could be understood if light were thought of as a stream of particles (photons) with energy equal to $h\nu$.



I got my Nobel prize for that.

Louis de Broglie (in 1923)

If light can behave both as a wave and a particle, I wonder if a particle can also behave as a wave?

Louis de Broglie

I'll try messing around with some of Einstein's formulae and see what I can come up with.

I can imagine a photon of light. If it had a "mass" of m_p, then its momentum would be given by

$$p = m_p c$$

where c is the speed of light.

Now Einstein has a lovely formula that he discovered linking mass with energy ($E = mc^2$) and he also used Planck's formula $E = hf$. What if I put them equal to each other?

$$mc^2 = hf$$

$mc^2 = hf$

So for my photon

$m_p = hf/c^2$

So if $p = m_p c = hf/c$

$p = m_p c = hf/c$

Now using the wave equation,

$c = f\lambda$ $(f = c/\lambda)$

So $m_p c = hc/\lambda c = h/\lambda$

$\lambda = h/p$

So you're saying that a particle of momentum p has a wavelength equal to Planck's constant divided by p?!

Yes!

$\lambda = h/p$

It will be known as the de Broglie wavelength of the particle

Confirmation of de Broglie's ideas

De Broglie didn't have to wait long for his idea to be shown to be correct.

In fact in 1929 I received a Nobel prize for my prediction of the wave nature of the electron.

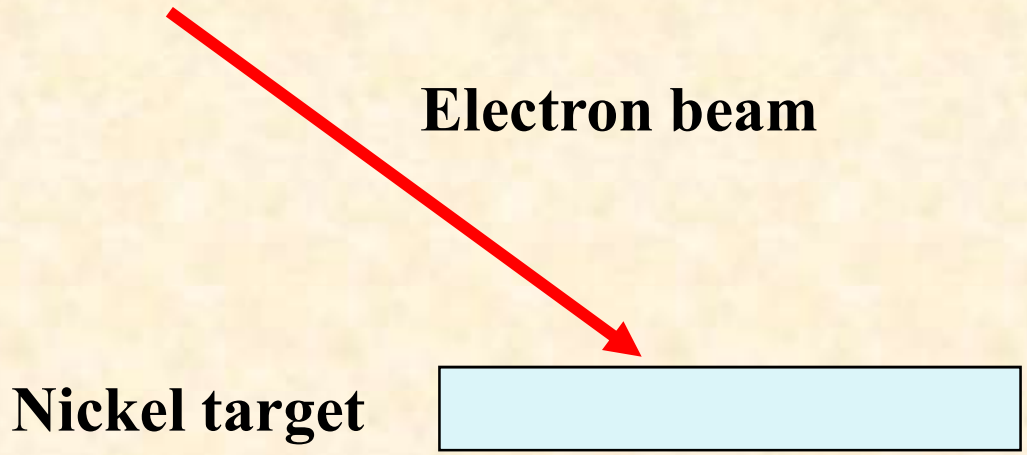
Confirmation of de Broglie

De Broglie's hypothesis was confirmed independently by Clinton Davisson (USA) and George Thomson (UK) in 1927

Ironically my Dad (J.J.) had won a Nobel prize for demonstrating that the electron was a particle!

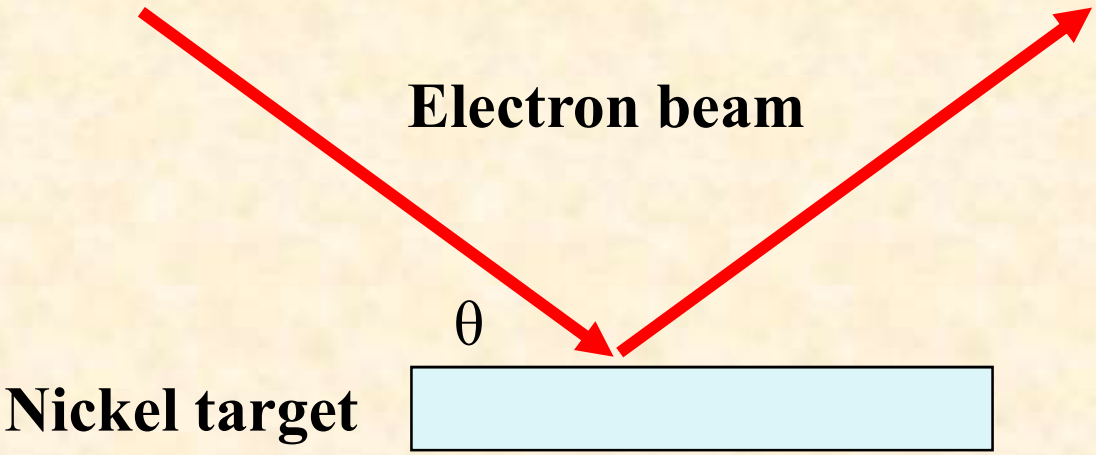
Electron Diffraction

Thomson and Davisson did similar experiments.
They fired a beam of electrons at a nickel target.



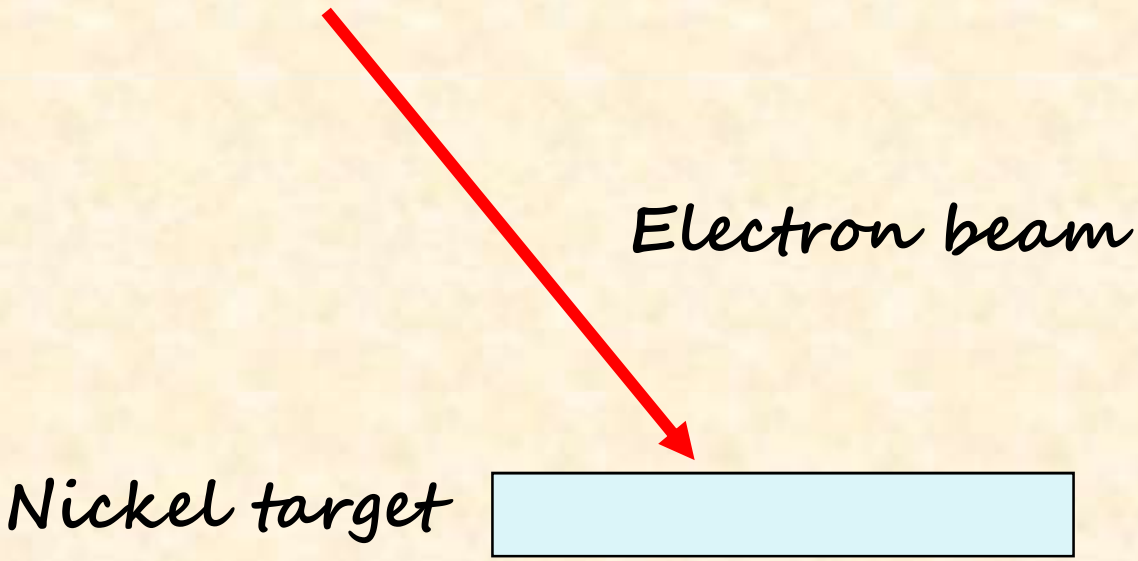
Electron Diffraction

They observed strong reflection at some angles,



Electron Diffraction

but not at others.



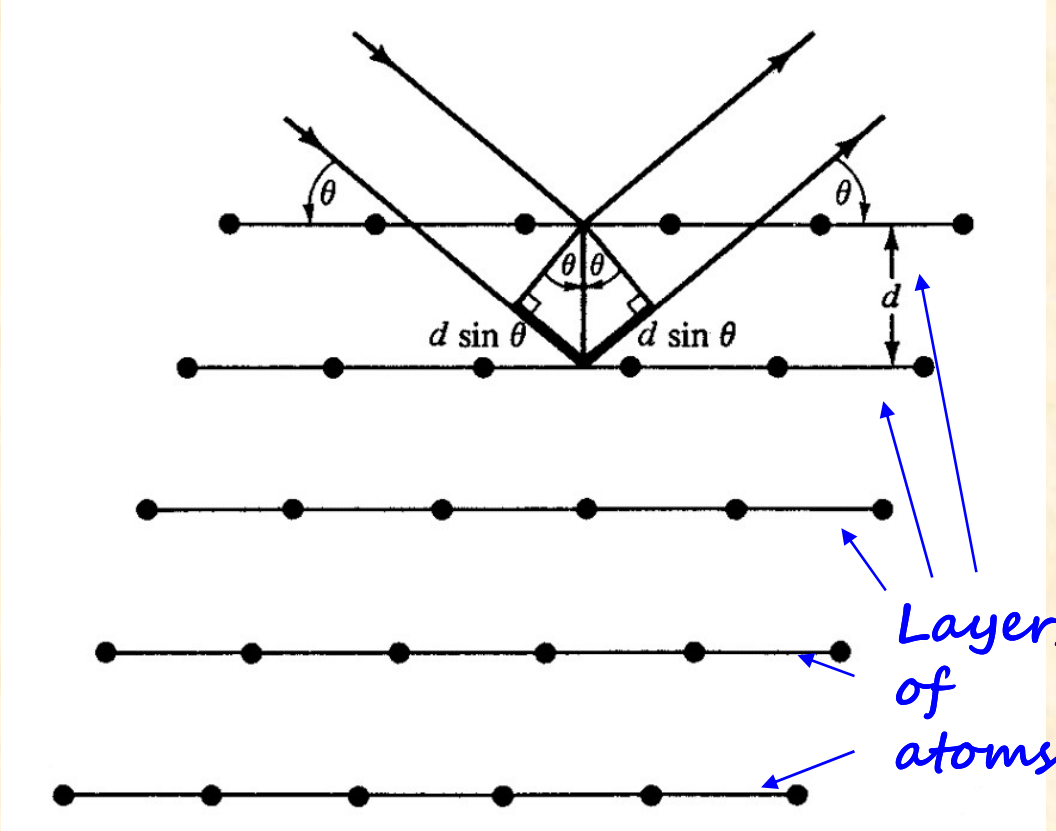
Electron diffraction

For constructive interference to occur between electrons reflected by different layers of atoms

Path Difference = $n\lambda$

$2d\sin\theta = n\lambda$

(Bragg formula used in 1914 by Bragg to study the diffraction of X-rays)



Electron Diffraction

They could find the crystal lattice separation from X-ray crystallography (using the Bragg formula), and then measure the wavelength of the incident electrons. The results agreed totally with de Broglie's predictions!

I knew they would!

Wave particle duality

We have seen that light can behave both as a wave (Young's double slit experiment) and a particle (Einstein's photo electric effect).

We have also seen that electrons can also behave as waves (electron diffraction predicted by de Broglie) and particles (J.J. Thomson)

Now which one is correct?

They both are!

It's called wave-particle duality.

Light can behave both as a wave and a particle, and electrons (and other "particles") can behave as both waves and particles.

But no experiment can ever show them behaving both as a wave and a particle at the same time! Remember it always!

Examples

What is the de Broglie wavelength of an electron travelling at 7×10^6 $m.s^{-1}$?

We know,

$\lambda = h/p = 6.63 \times 10^{-34} / 9.11 \times 10^{-31} \times 7 \times 10^6$

$= 1 \times 10^{-10}$ m (more or less)

This is similar to the average spacing between atoms in a crystal.

Examples

What is the de Broglie wavelength of a tennis ball (mass 58g) travelling at 10^2 m.s^{-1}?

Here, $\lambda = h/p = 6.63 \times 10^{-34}/0.058 \times 10^2$

$= 1 \times 10^{-34}$ m (more or less)

The tennis ball would have to interact with something of a similar size to demonstrate any wave properties!

Remember the nucleus of an atom is around 10^{-15} m, a million, million, million times bigger than this!

A harder example

Electrons are accelerated through a p.d. of 54 V and are directed at a Beryllium crystal with a spacing between atoms of 7.38×10^{-9}m. Calculate the de Broglie wavelength and the first order (n=1) angle of diffraction.

Solution

Energy of electrons = $eV = mv^2/2$

$v = (2eV/m)^{1/2}$

$p = mv = (2eVm)^{1/2}$

$\lambda = h/p = h/(2eVm)^{1/2} = 1.67 \times 10^{-10}$ m

Assuming that this behaves as a diffraction grating (like light passing through many slits)

$d\sin\theta = n\lambda$

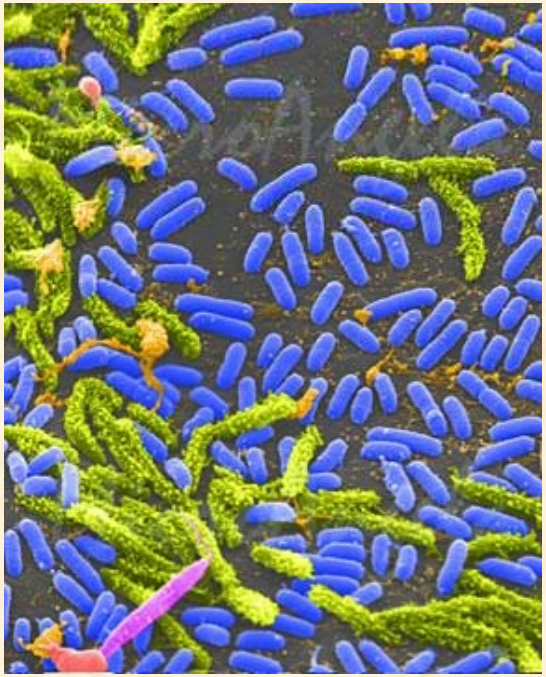
$\sin\theta = n\lambda/d = 1 \times 1.67\ 10^{-10}/7.38 \times 10^{-9} = 2.28 \times 10^{-2}$

Hence $\theta = 1.3°$

Application : Electron Microscope

This uses the wave nature of electrons to produce pictures of very small objects, too small to be imaged using visible light (which only has a wavelength of around 500 nm compared with electrons with a wavelength f around 0.1 nm).

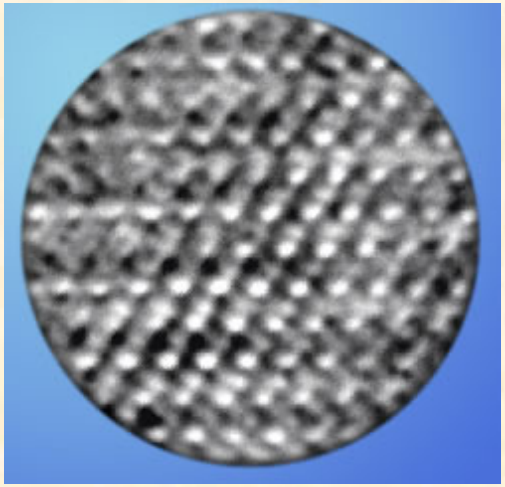
Electron Microscope Pictures



1µm

Just look at the resolution !

0.1nm



7.5µm

So that's it !

Cool stuff eh?

Let's try at least some questions from CU 10 years question papers!