

Short Answers Type Questions on Quantum

- Ψ itself has no physical meaning but a function of it represents a physically meaningful quantity. How?
 $\Psi^* \Psi = \rho$, ρ is the probability density of finding the particle.
- $\Delta x \Delta p_x \geq h/(4\pi)$ is one of the uncertainty relation. What is the other one?
 $\Delta E \Delta t \geq h/(4\pi)$
- In the famous Schrodinger equation $H\psi = E\psi$. What is H ?
The Hamiltonian, the total energy (kinetic+ potential) operator.
- In the famous Schrodinger equation $H \Psi = E \Psi$; divide both sides by Ψ to get $H = E$. Possible?
No, H is not multiplicative but differential;
- Energy is often expressed in terms of a quantity whose SI unit is m^{-1} . What is that quantity?
Wave number; $1/\lambda$
- How does wave number ($1/\lambda$) represent energy?
 $E = h\nu = hc/\lambda$ or, $E/hc = 1/\lambda$
- $(mc^2 = p.c) = E = (h\nu = h c/\lambda)$
 $p.c = h c/\lambda$; **$p = h/\lambda$** , all in 1905; can you call it de Broglie relation?
For light only (in 1905); de Borglie's one is a general one.
- The first experiment that was solved by a new revolutionary theory. What is that experiment that classical science failed to explain?
Black body radiation
- The smallest possible value of a universal constant is named after the name of a physicist. Who is he/she?
M. Planck; $h = 6.626 \times 10^{-34} \text{ J.s}$
- In explaining the BBR, what new concept was introduced which has no classical analogue?
Quantization of energy emission
- N. Bohr, in his theory of H-atom used the new concept of *quantization*. Quantization of what quantity?
Angular momentum of e^-
- In his theory of H-atom, what Bohr *could* and could *not explain* of an atom?
IP and line spectra. Finer spectra of multi electron atom.
- Einstein published *five* papers in the same journal, in the same year. One of those papers brought him the Nobel. On which *work* it was? In which year were they published?

1905; PEE

14. A simple looking equation brought de Broglie his Nobel prize. What is that equation and what's it all about?

$P = h/\lambda$; wave-particle duality in general.

15. **$g = Af$** , A is operator, f is function of x . If **$A = \exp(\)$** and **$f(x) = \ln x$** , then **g** is?

$e^{\ln x} = P$, $\ln P = \ln x$; Or, $P = x$

16. If **$A f(x) = 3x^2 f(x) + 2x df/dx$** , where A is an operator and f is an arbitrary function. What is A ?

$A = 3x^2 . + 2x (d/dx) .$

17. If **$A e^x = e^x$** ; give *three different* operators A .

$A \rightarrow 1, \quad d/dx \quad d^2/dx^2$

18. What do you suppose is meant by the *zeroth* power of an operator?

$A^0 = 1$. Applying A zero times to f leaves f unchanged.

19. Under what condition, **$(A + B)^2 = A^2 + 2AB + B^2$** ? [A and B are operators].

$AB = BA$

20. Under what condition, **$A(B + C) = AB + AC$** ? [A, B , and C are operators].

They are linear operators.

$A(B+C)f = A(Bf + Cf)$ [sum rule for B & C]

$= A(Bf) + A(Cf)$ [A is linear]

$= (AB)f + (AC)f$ [product rule]

$= (AB + AC)f$ [sum rule for AB & AC .]

21. Complex conjugation **$(f + g)^*$** is linear operation. Justify or criticize.

Not justified. A is linear if (i) $A(f + g) = Af + Ag$ and (ii) $A(cf) = c(Af)$

$(f + g)^* = f^* + g^*$ but $(cf)^* = c^* f^* \neq c f^*$ (i) is satisfied but not (ii).

22. A is linear if

(i) **$A(f + g) = Af + Ag$** and (ii) **$A(cf) = c(Af)$** . Give example where (ii) is satisfied but not (i).

$()^{-1} (d/dx) ()^{-1}$

23. Give the kinetic energy operator T for a two particle, three dimensional system.

$T = -(\hbar^2/2m_1)(\partial^2/\partial x_1^2 + \partial^2/\partial y_1^2 + \partial^2/\partial z_1^2) - (\hbar^2/2m_2)(\partial^2/\partial x_2^2 + \partial^2/\partial y_2^2 + \partial^2/\partial z_2^2)$

24. For the PIB (cubic), what is the degeneracy of the energy level with **$8mL^2E/h^2 = 12$** .

Non-degenerate or $g = 1$

25. For the PIB (cubic), what is the degeneracy of the energy level with **$8mL^2E/h^2 = 14$** .

$g = 6$

1 2 3

1 3 2

2 1 3

2 3 1

3 1 2

3 2 1

26. **$C_1 \Psi_{151} + C_2 \Psi_{333} + C_3 \Psi_{511}$**

This combination of the PIB stationary state wave functions is also an eigen function of PIB Hamiltonian. Justify or criticize.

Yes. Ψ_{151} , Ψ_{333} and Ψ_{511} are degenerate states.

27. If the state function $\Psi = (2/L)^{1/2} \sin(3\pi x/L)$ for a PIB, what is the possible value that results as measurement of energy ?

It is exactly the eigen value since Ψ is an eigen function for the PIB.

And the measured value is $E_3 = 9h^2 / (8mL^2)$.

28. we will *not* get one of the eigen values of operator B when we measure the property B and the system's state function is not an eigen function of B . Justify or criticize.

We will get; only we can't predict which eigen value.

29. For the $n=25$ SHO wave function, the sign of Ψ in the right-hand classically forbidden region is opposite the sign in the left-hand classically forbidden region. Justify or criticize.

Ψ_{25} is odd function. Ψ_0 is even, and if Ψ_n is odd, Ψ_{n+1} is even;

Ψ does not oscillate in the forbidden regions.

30. Adding a constant C to the potential energy leaves the stationary state wave functions unchanged and simply adds the energy eigenvalues.

Justify or criticize.

Justified. $(H + C) \Psi = H \Psi + C \Psi = E \Psi + C \Psi = (E + C) \Psi$.

31. $(\Delta A)^2$ is defined as $\langle (A - \langle A \rangle)^2 \rangle$. Hence $(\Delta A)^2 = \langle A^2 \rangle - \langle A \rangle^2$

justify or criticize.

Justified.

$$\begin{aligned} (A - \langle A \rangle)^2 \Psi &= (A - \langle A \rangle) (A - \langle A \rangle) \Psi \\ &= A^2 \Psi - A \langle A \rangle \Psi - \langle A \rangle A \Psi + \langle A \rangle^2 \Psi \\ &= A^2 \Psi - 2 \langle A \rangle A \Psi + \langle A \rangle^2 \Psi \quad \text{or,} \\ \langle (A - \langle A \rangle)^2 \rangle &= \langle A^2 \rangle - 2 \langle A \rangle \langle A \rangle + \langle A \rangle^2 = \langle A^2 \rangle - \langle A \rangle^2 \end{aligned}$$

32. Under what condition $\langle c f | B | g \rangle = \langle f | B | c g \rangle$?

$c = c^*$; B is linear.

33. What operator is shown to be Hermitian by the equation $\langle f | g \rangle = \langle g | f \rangle^*$?

The operator is *multiplication by 1*.

34. If A and B are Hermitian *then* under what condition, (AB) is Hermitian?

A and B would commute.

Proof: $\langle \Psi | AB | \Psi \rangle = \langle A \Psi | B | \Psi \rangle = \langle (BA) \Psi | \Psi \rangle$; For (AB) to be Hermitian,

$\langle \Psi | AB | \Psi \rangle = \langle (AB) \Psi | \Psi \rangle$; Comparing the equations, $AB = BA$

35. If A and B are Hermitian then, is their anticommutator $[AB + BA]$ also Hermitian?

Yes.

Proof: $\langle \Psi | AB + BA | \Psi \rangle = \langle \Psi | AB | \Psi \rangle + \langle \Psi | BA | \Psi \rangle$

$= \langle BA \Psi | \Psi \rangle + \langle (AB) \Psi | \Psi \rangle = \langle (BA + AB) \Psi | \Psi \rangle$

36. If $(x p_x)$ Hermitian?

No.

Both x and p_x are Hermitian; but they don't commute. So, $(x p_x)$ is not Hermitian.

37. If $(1/2)(x p_x + p_x x)$ Hermitian?

38. Yes.

Both x and p_x are Hermitian, so their anticommutator $(x p_x + p_x x)$ is Hermitian.