Nuclear stability

Obvious question: why some nuclides are stable and some are radioactive?

Even-odd nature of protons and neutrons.

Z	N	A Z+N	No. of stable nuclides	Examples
Even	Even	Even	165	
Even	Odd	Odd	55	
Odd	Even	Odd	50	
Odd	Odd	Even	4 (or 5)	^{2}H, ^{6}Li, ^{10}B, ^{14}N, (^{180}Ta)

Number of stable nuclides are maximum when both N and Z are even, suggesting tendency to form n-n and p-p pairs for stability, like electron pairs. The composition of earth's crust about 85% is consisted of even-N even-Z nuclides, like ^{16}O, ^{28}Si, ^{56}Fe, ^{40}Ca, ^{24}Mg, about 13% of odd Z even N nuclides like ^{27}Al, ^{23}Na, ^{39}K etc.

The no. of stable nuclides having either Z or N odd is approx. 1/3 of those where both are even. The number of stable nuclides having odd Z and odd N are almost equal (both cases A is odd).

The tendency of proton to pair with a proton and neutron with neutron is well observed in distributing stable nuclides amongst light nuclides (elements from O to Cl). The rule for forming stable nuclides over this range is successive addition, strictly, of two neutrons, one at a time, followed by two protons, one at a time. And then again the turn of neutrons, and so on. This results in the formation of 3 isotopes of elements of even Z as A, A+1, and A+2, but only one isotope of elements of odd Z.

The violation of the sequence of n+n, p+p, n+n,…. Results in formation of radioactive nuclide.

Beyond $_{35}$Cl, rule of nucleon addition becomes more complicated.

It may also be noted that out of 20 stable mono-isotopic nuclides, 19 are odd Z. the sole exception is ^{9}Be.

The isotopes having no stable isotopes are also of odd Z, those being $_{43}$Tc, $_{61}$Pm and also elements beyond 83Bi ($Z \geq 84$).

Atomic nucleus

Composition of nucleus

The structure of the nucleus has not been fully understood even upto present time. We will discuss only the general observations and some qualitative discussions of various models and interpretations suggested towards its structure and stability.

Analysis of positive rays from different elements ultimately established that proton is a common constituent of all nuclides. Long ago, proton electron hypothesis suggested that nuclei should contain enough proton to account for its mass. A nucleus whose mass is close to the whole number A (in amu) should contain A protons. Since actual charge of the nucleus is +ze, where Z is the atomic number, the nuclei should contain (A-Z) no. of electrons to neutralize the excess positive charge. The number A has been called the mass number and is the integer closest to the atomic mass (in amu).

The proton electron hypothesis was apparently consistent with emission of alpha, beta particles from radioactive nuclides. Beta are identical with electrons. And alpha could be combination of four protons and two electrons. Any one of them may be ejected under suitable conditions.

However, there soon appeared a no. of strong arguments against the possible existence of electrons in nucleus. E.g. Uncertainty principle and nuclear size, nuclear spin, nuclear magnetic moment, etc.

The discovery of neutrons by Chadwick (1932) made a significant contribution in pacifying the problem of nuclear composition. Its mass, half integral spin, and electrical neutrality fitted with the observed properties of nucleus.

It is now believed that neutron and protons are the constituent of nucleus. The nuclear constituent particles are known as nucleons. They follow Fermi-Dirac statisitics (Fermions).

Shape of nucleus

For all practical purposes, nucleus is considered to be spherical. If the charge distribution within a nucleus is not spherically symmetrical, the nucleus will have an electric quadrupole moment. Interaction of this moment with the orbital electrons will give rise to slight changes in the atomic energy levels, resulting in hyperfine splitting of spectral lines. By careful observations, it is possible to distinguish such hyperfine splitting from those caused by nuclear magnetic moment. It has been found that nuclei whose spin quantum numbers are 1 or more do deviate slightly from sphericity.

Nuclear size

Scattering of alpha particle experiment provided an indea about the size of nucleus. In that experiment suppose an α particle of mass M_α and velocity v approaches the nucleus of charge Ze. The charge on the α particle is 2e. The closest distance of approach (d_0) in a head-on collision is determined at the point where the α particle reverses its direction. At this point the kinetic energy of the α particle is just balanced by the potential energy due to repulsion.

$$\text{Force of repulsion} = \frac{2e\,Ze}{4\pi\varepsilon_0 d_0^2} = \frac{Ze^2}{\pi\, 2\varepsilon_0 d_0^2}$$

$$\therefore \text{ Corresponding potential energy} = \frac{Ze^2}{2\pi\varepsilon_0 d_0}$$

$$\text{At the point of closest approach, } \frac{Ze^2}{2\pi\varepsilon_0 d_0} = \frac{1}{2}M_\alpha v^2, \text{ or, } d_0 = \frac{Ze^2}{\pi\varepsilon_0 M_\alpha v^2}$$

$$\left[\text{In C.G.S. units, } \frac{2Ze^2}{d_0} = \frac{1}{2}M_\alpha v^2 \,;\, d_0 = \frac{4Ze^2}{M_\alpha v^2}\right]$$

α particle from natural sources have velocities 1.3-1.9 X 10^7 m/s. this gives the value of d_0 around 5 X 10^{-15} m, or 5 fm (femtometer, 10^{-15} m) for aluminium.

Scattering of fast neutrons give more dependable information on the radius R of a nucleus as the distance from the nuclear centre within which the nuclear forces act.

From a number of experiments it has been found that volume of a nucleus is directly proportional to the mass no. A, i.e. number of nucleons present.

$4/3\ \pi R^3\ \alpha\ A$

$R^3\ \alpha\ \frac{3A}{4\pi}$

$R = R_0 A^{1/3}$

Where, R_0 is some constant. It has a value of 1.4 fm.

The radius of Al nucleus was estimated to be 4.2 fm. Similar results are obtained from scattering of high energy electrons. So, nucleus is very small compared to the atom as a whole.

Nuclear density

The radius of the nucleus is proportional to $A^{1/3}$, where A is the mass number. So, assuming spherical shape, the volume of the nucleus is given by $4/3\pi R^3$, and it should be proportional to A. this leads to an important conclusion that nuclear volumes are proportional to nuclear masses and all nuclei should have approximately same nuclear density.

Since almost all the mass of an atom is located in the nucleus, the density of nucleus is very high, it may be 10^{11}Kg/cc.

Mass defect and Packing Fraction

The mass of an atom is almost entirely due to its nucleus. With the discovery of isotopes and Aston's mass spectrograph, it was found that great majority of the elements consisted of isotopic nuclides, i.e. their atomic weights were nearly integers. Aston noticed that in most cases atomic masses deviated slightly from the nearest whole numbers (sum of protons and neutrons) representing the mass no. the magnitude of this deviation varies from nucleus to nucleus.

Aston suggested that the true atomic weights of elements are whole numbers. Fractional atomic weights are explained by existence of isotopes of varying abundance. This was practically the revival of Prout's hypothesis proposed a hundred years ago that all elements are made up of hydrogen. It was soon realised that though the isotopic masses were close to whole numbers, they always differ slightly from whole numbers. Aston generalised this deviation by so called 'Mass Defect' and 'Packing Fraction'.

The nearest integer to the mass number of a nuclide (in atomic mass unit scale) was called the mass number of that isotope. The difference between the actual isotopic mass and the mass number was termed as Mass Defect of that nuclide.

At present, the mass no. of any isotope may be looked upon as the total no. of protons and neutrons in it (neutrons were discovered in 1932, Aston proposed this term in 1927). Accordingly, deviation of isotopic masses from whole numbers, the so called mass defect may be understood from mass energy interconversion. Mass defect is explained as mass excess ΔM which may be positive or negative. For a nuclide of mass M and mass number A, we may write

$$M = A + \Delta M \quad \text{or,} \quad \Delta M = M - A$$

i.e., **Mass Excess** ('Mass Defect') = **Isotopic Mass − Mass Number.**

Aston introduced a term "Packing Fraction" for each nuclide to compare their mass excess. It was defined as

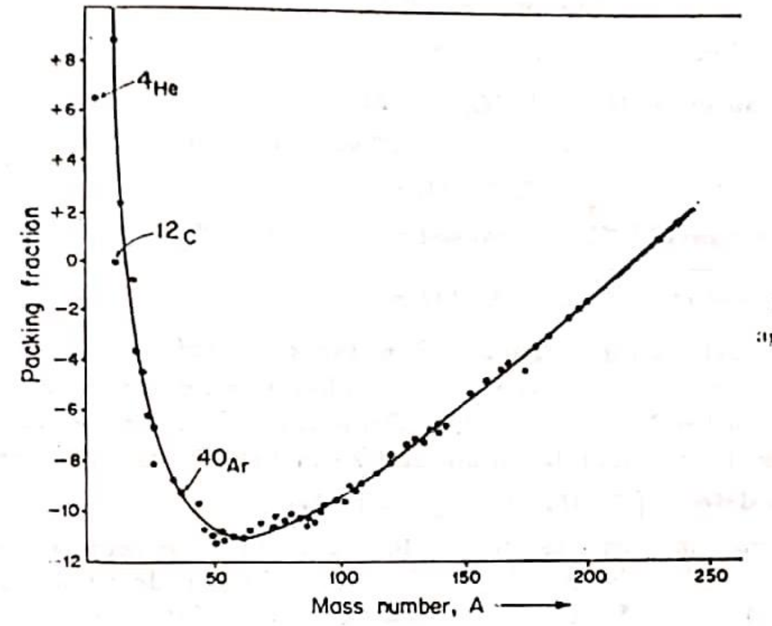
$$\textbf{Packing Fraction (f)} = \frac{\textbf{Isotopic Mass } (M) - \textbf{Mass Number } (A)}{\textbf{Mass Number } (A)} \times 10^4 = \frac{\Delta M}{A} \times 10^4.$$

The factor of 10^4 was used to obtain figures which are easy to plot.

The PF (Packing Fraction) value may be positive, negative zero.

PF gives an indication about the stability of the nucleus.

A negative value of PF indicates that isotopic mass is actually less than nearest whole number (ΔM negative). So, a small fraction of mass has been converted into energy of formation of that nucleus. If we want to split the nucleus, this amount of energy has to be supplied. So, more negative PF of an element, more stable should be the nucleus. Again, a positive PF value means the nucleus should be less stable.



Plot of packing fraction against mass no.

From the plot, the observations may be

- The PF of stable nuclei lie more or less on a smooth curve.
- The curve passes through minima in the mass no. range 50-60 (transition elements Cr, Fe, Co, Ni) and such nuclides are more stable ones and are highly abundant in nature. The curve becomes positive at about Hg (A ~ 175) indicating instability of heavier nuclides.

- Generally the elements with higher positive PF tend to move to the region where PF value is minimum (i.e. most negative). This is achieved by the lighter elements through fusion and heavier elements by fission.
- There is a bifurcation of the curve for lighter elements. One branch contains The ^{4}He, ^{12}C, ^{16}O and other branch occupies other nuclides from ^{1}H to ^{20}Ne. The ^{4}He, ^{12}C ^{16}O whose mass nos. are integral multiples of 4 (i.e. mass no. of alpha particle) are relatively more stable. In fact, these nuclides are much resistant to nuclear disintegration compared to other neighbours. That is why, alpha particle was considered to be an efficient subsidiary unit in constructing lighter nuclides.
- Apparently, nuclides with zero or positive PF values should not be stable, but some lighter nuclides (like He) with positive PF value are stable. This problem arises due to consideration of mass no. instead of actual mass of the nucleons. The actual mass of the nucleons is always greater than the mass no. e.g. He mass is found to be 4.003 amu. But actual mass of He nucleus is 2X1.0076 + 2X1.0089 = 4.033 amu.

So, mass loss is 0.03 units. PF = $\frac{4.003-4.033}{4}$X10^4 = -75.

The actual expression of PF should be, PF = $\frac{nuclear\ mass - mass\ of\ nucleons}{Mass\ no.}$ X 10^4

Nuclear Binding Energy (NBE)

After the discovery of neutrons the mass of any atom may estimated in terms of the masses of the constituent electrons, protons, and neutrons. These quantities are obtained from mass spectrometric studies with the precision of the order of few parts per millions.

However, the estimated (theoretically calculated) mass of a nuclide was always found to be greater than precisely determined experimental mass of that isotope. The differences between the mass of the constituents of the atom and experimental atomic mass may be called the true mass defect of a nuclide (isotope).

For ${}^{A}_{Z}X$

Atomic mass = $(A-Z)m_n + Zm_p + Zm_e$ where m_n, m_p, m_e are the masses of neutron, proton and electron, respectively

But, measured atomic mass (M) is less than theoretically calculated value.

True mass defect = $[(A\text{-}Z)m_n + Zm_p + Zm_e] - M$

$= (A\text{-}Z)m_n + Zm_H - M$ where m_H = mass of a hydrogen

= mass of proton + mass of electron

$= m_p + m_e$

This lost mass may be considered as the mass which has been converted to the energy released in the formation of nucleus from individual protons and neutrons. [since the true mass of an atomic species has been compared with the estimated mass, the mass of the electrons have to be considered. That is why, we considered the mass of hydrogen atom, not that of the proton alone, in calculating true mass defect. However, we have neglected the very small change in mass accompanying the formation of hydrogen atom from a proton and an electron. The binding energy of an electron to the nucleus is also neglected].

The magnitude of binding energy may be estimated from the true mass defect of a nuclide using the mass energy equivalence $E = \Delta mc^2$ [where, E = energy released, Δm = mass lost, and c = velocity of light in vacuum].

= 931 MeV

Binding energy corresponding to 1 amu mass loss is 931 MeV.

So, binding energy of a nucleus (B) = $931 \times [(A-Z)m_n + Zm_p + Zm_e - M]$ MeV

The average binding energy per nucleon ($\overline{B}$) is obtained by dividing the binding energy by the no. of nucleons present.

$$\overline{B} = \frac{total\ binding\ energy\ of\ the\ nucleus}{total\ number\ of\ nucleons} = \frac{B}{A}$$

Example I: The helium atom contains 2 protons, 2 neutrons and 2 electrons. The mass of 2 protons and 2 electrons should equal the mass of 2 hydrogen atoms (^1_1H). Hence,

$$M_{He} = 2M_H + 2M_n. \text{ (Subscripts refer to respective species).}$$

In atomic mass units we have : $M_H = {}^1_1H = 1{\cdot}007825$; $M_n = {}^1_0n = 1{\cdot}008665$.

$$\therefore \quad M_{He} = 2 \times (1{\cdot}007825 + 1{\cdot}008665) \qquad = 4{\cdot}032980 \; m_u.$$

But the mass spectrometric mass of the helium atom $= 4{\cdot}002603 \; m_u.$

Mass defect $= 0{\cdot}030377 \; m_u$

Nuclear binding energies are often expressed in the non-SI unit of MeV (mega-electronvolt). The electron-volt (eV) is the energy gained by an electron when accelerated through a potential difference of one volt. Since the charge of an electron (e) is $1{\cdot}602\times10^{-19}$ C,

$$1\text{eV}=1{\cdot}602\times10^{-19}\ \text{C V}=1{\cdot}602\times10^{-19}\ \text{J}.$$

The binding energy corresponding to 1 atomic mass unit, i.e. m_u, would be equal to

$$1{\cdot}660\times10^{-27}\ \text{kg}\times(2{\cdot}9979\times10^{8}\ \text{m s}^{-1})^{2}$$

$$=14{\cdot}9191\times10^{-11}\ \text{J}.$$

$$=931{\cdot}28\ \text{MeV}.$$

For most calculations, 931 MeV is a good approximation. So, the binding energy of any nucleus in MeV is equal to the true mass defect in atomic mass unit or m_u multiplied by 931.

A plot of av.NBE against mass no.gives a nearly smooth curve which is practically inversion of Packing Fraction vs mass no. curve.

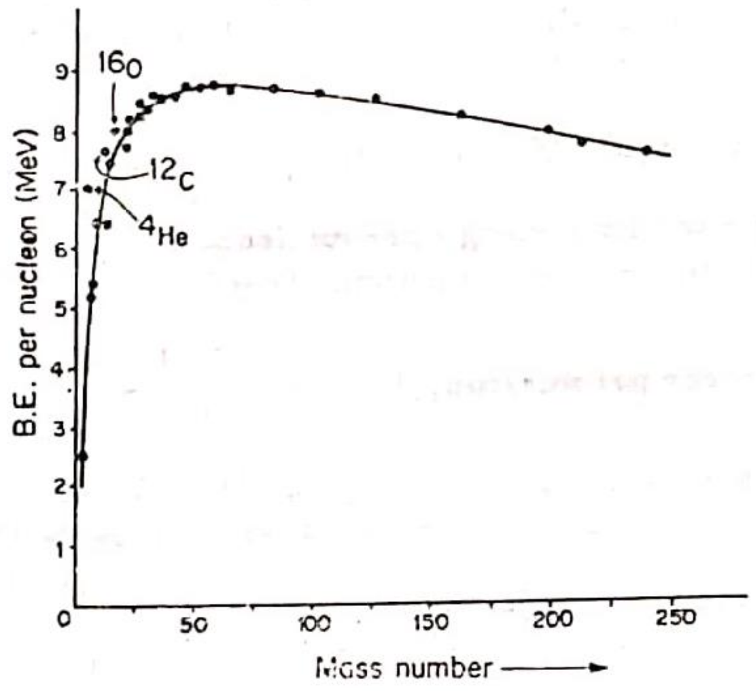
Maximum NBE indicates that in the mass no. range 50-60 the atoms are stable.

The binding energy of ^{4}He, ^{16}O, and ^{12}C are exceptionally high indicating they are highly stable.

It may be observed that mean av.NBE is about 8 MeV over a considerable range of mass no.

With the increase in mass no., NBE first increases with increase in mass no. this suggests that energy will be released if one could fuse lighter nuclei together into more stable ones. This forms the basis of nuclear fusion.

In the high mass no. range (say, about 100) NBE decreases sharply with mass no. showing heavier nuclei are less stable. Energy would be released if one could split heavier nuclides into some lighter stable fragments. This is the basis of nuclear fission process.



The nature of curve obtained by plotting average binding energy per nucleon against mass no.

The maxima for average NBE per nucleon varies periodically for the values of A which are integral multiples of 4. these nuclides are ^{4}He, ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, ^{28}Si which are very much stable. Only ^{8}Be is an exception and it actually disintegrates into two alpha particles.

Thus nuclides constructed by alpha particles as blocking units are more stable than the neighbouring ones. (Rule od oddo).

There is a rise in average NBE per nucleon from 8.0 MeV (A = 16) to about 8.30 MeV (A = 28-32) and then it remains almost constant for medium nuclides (A = 30-90) ($\overline{B} = 8.5 \pm 0.2$ MeV). This zone gives the largest zone of stability. This value is considered to be the diagnostic parameter of predicting the stability of nucleus.

After Zr, $\overline{B}$ falls gradually from ~8.7 MeV to ~7.5 MeV at A~210. upto A~210, the nuclides are stable, but after the limit the nuclides become very unstable. In fact $^{209}Bi_{83}$ is the heaviest nuclide known to be stable.