Asymptotes

Let the n th degree polynomial curve is expressed as

$$x^n \varphi_n\left(\frac{y}{x}\right) + x^{n-1}\varphi_{n-1}\left(\frac{y}{x}\right) + \cdots + \varphi_1\left(\frac{y}{x}\right) = 0$$

assuming it passes through origin.

We write the equation of asymptote as

$$y = mx + c$$

then $\frac{y}{x} \to m$ as $x \to \infty$ and $\varphi_n(m) = 0$ provides as it's real roots the potential gradient of all asymptotes.

Now $\frac{y - mx - c}{\sqrt{1+m^2}} = d \to 0$ as $x \to \infty$ gives

$$\varphi_n\left(m + \frac{c+u}{x}\right) + \frac{1}{x}\varphi_{n-1}\left(m + \frac{c+u}{x}\right) + \cdots$$

on Taylor's series expansion gives

$$\frac{c+u}{x}\varphi_n'(m) + \frac{1}{x}\varphi_{n-1}(m) + \frac{c+u}{x^2}\varphi_{n-1}'(m) + \frac{1}{x^2}\varphi_{n-2}(m)$$
$$+ \cdots = 0$$

Since $\varphi_n(m) = 0$ and $u \to 0$ as $x \to \infty$.

Multiplying throughout by x, we easily get

$$c = -\frac{\varphi_{n-1}(m)}{\varphi_n'(m)}.$$

For repeated root, i.e. $\varphi_n'(m) = 0$ and $\varphi_{n-1}(m)$, we get similarly, the eqn.

$$\frac{c^2}{2}\varphi_n''(m) + c\,\varphi_{n-1}(m) + \varphi_{n-2}(m) = 0$$

for the two values of c with same m.