Integrator Circuit using R and C :

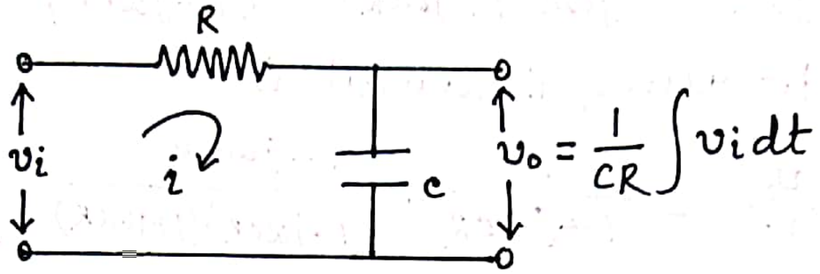


Fig-1.

The R-C low pass filter or R-C integrator circuit is shown in fig-1.

The circuit equation is $v_i = \frac{1}{C}\int i\,dt + iR$

$$\Rightarrow iR = v_i - \frac{1}{C}\int i\,dt$$

$$\Rightarrow i = \frac{v_i}{R} - \frac{1}{CR}\int i\,dt$$

If the time constant (CR) of the circuit is very large compared to the time period (T) of the input signal then the second term is neglected and hence

$$i = \frac{v_i}{R} \text{ when } CR >> T$$

The output voltage is taken across the capacitor C and is given by

$$v_o = \frac{1}{C}\int i\,dt = \frac{1}{CR}\int v_i\,dt$$

Again the total impedance of the circuit (Fig-1) is

$$Z = R + \frac{1}{j\omega C} = \frac{1 + j\omega CR}{j\omega C}$$

The current

$$i = \frac{v_i}{Z} = \frac{v_i \times j\omega C}{1 + j\omega CR}$$

Therefore the output voltage developed across C is

$$v_o = \frac{i}{j\omega C} = \frac{v_i \times j\omega C}{j\omega C(1+j\omega CR)} = \frac{v_i}{(1+j\omega CR)}$$

The transfer gain of the circuit is

$$H(j\omega) = \frac{v_o}{v_i} = \frac{1}{1+j\omega CR} = \frac{1-j\omega CR}{(1-j\omega CR)(1+j\omega CR)} = \frac{1-j\omega CR}{(1+\omega^2C^2R^2)}$$

$$\therefore H(j\omega) = \frac{v_o}{v_i} = \frac{1}{(1+\omega^2C^2R^2)} - j\frac{\omega CR}{(1+\omega^2C^2R^2)}$$

$$\therefore \left|\frac{v_o}{v_i}\right| = \frac{1}{(1+\omega^2C^2R^2)^2} + \frac{\omega^2C^2R^2}{(1+\omega^2C^2R^2)^2} = \frac{1}{1+\omega^2C^2R^2}$$

$$\therefore \left|\frac{v_o}{v_i}\right| = \frac{1}{1+\left(\frac{\omega}{\omega_c}\right)^2} = \frac{1}{1+(f/f_c)^2}$$

where $\omega_c = \frac{1}{CR} = 2\pi f_c$, f_c is called the cut-off frequency

$$\boxed{\therefore f_c = \frac{1}{2\pi CR}}$$

The transfer characteristics of the R-C low pass filter or R-C integrator circuit is shown in Fig-2

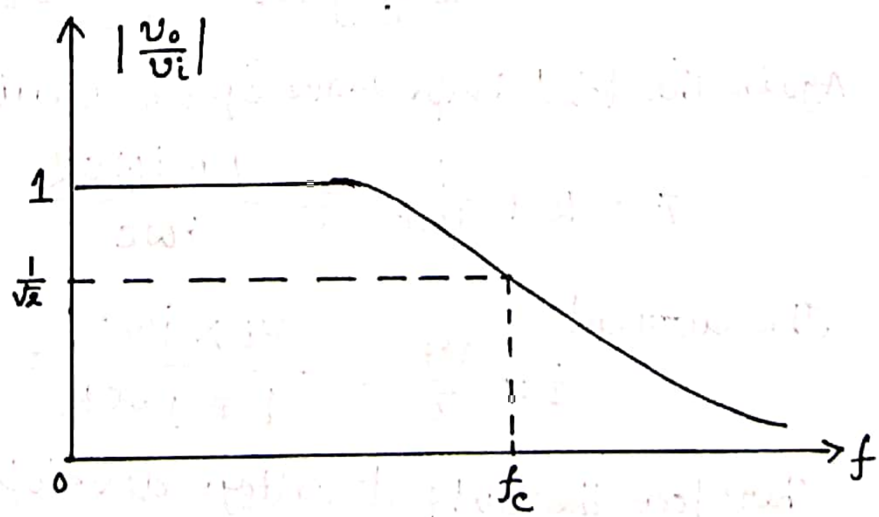


Fig-2

Digital to Analog Converter or D/A Converter :-

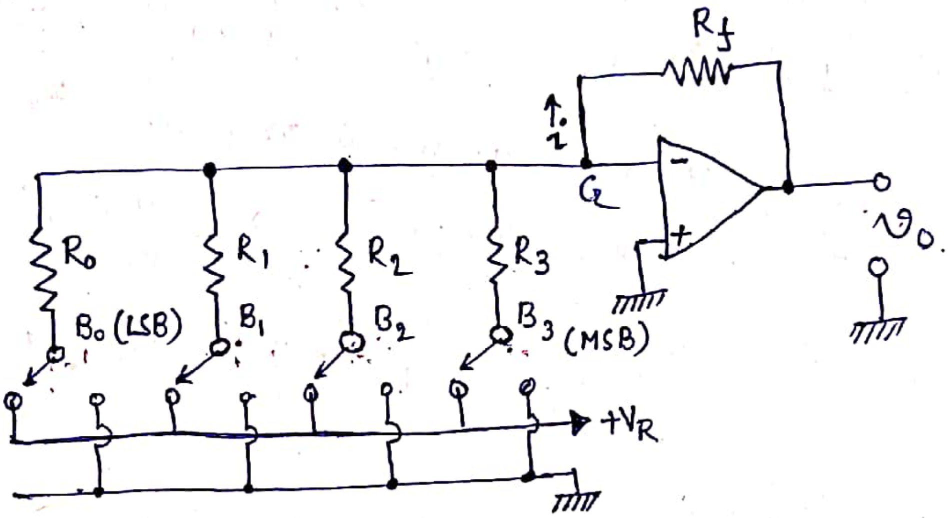


Fig - 1

We consider the Conversion of a 4-bit digital word $(B_3 B_2 B_1 B_0)$ into an analog form. The decimal equivalent (N) of a 4-bit digital word $(B_3 B_2 B_1 B_0)$ is

$$N = 2^3 B_3 + 2^2 B_2 + 2^1 B_1 + 2^0 B_0 \quad (B_i = 0 \text{ or } 1) \qquad \text{---} (1)$$

where each bit of the word Contributes to the final value with a weight 2^i multiplied by the value of B_i $(i = 0, 1, 2, 3)$. Since B_i is either 0 or 1, the Contribution is clearly zero or the bit weight. Here B_0 is the least significant bit (LSB) and B_3 is the most significant bit (MSB). B_i is 1 means all the resistors are connected to $+V_R$. B_i is 0 means all the resistors are connected to ground potential. The resistors R_0, R_1, R_2 and R_3 in the circuit are weighted so that their successive resistors ratio is 2 i.e $\frac{R_0}{R_1} = \frac{R_1}{R_2} = \frac{R_2}{R_3} = 2$.

Let $R_0 = \frac{R}{2^0}$, $R_1 = \frac{R}{2^1}$; $R_2 = \frac{R}{2^2}$; $R_3 = \frac{R}{2^3}$

$\therefore$ $R_0 = R$, $R_1 = \frac{R}{2}$; $R_2 = \frac{R}{4}$ and $R_3 = \frac{R}{8}$.

The current i to the inverting terminal is

$$i = V_R\left(\frac{B_3}{R_3} + \frac{B_2}{R_2} + \frac{B_1}{R_1} + \frac{B_0}{R_0}\right).$$

$$= V_R\left(\frac{2^3 B_3}{R} + \frac{2^2 B_2}{R} + \frac{2^1 B_1}{R} + \frac{2^0 B_0}{R}\right)$$

$$= \frac{V_R}{R}\left(2^3 B_3 + 2^2 B_2 + 2^1 B_1 + 2^0 B_0\right)$$

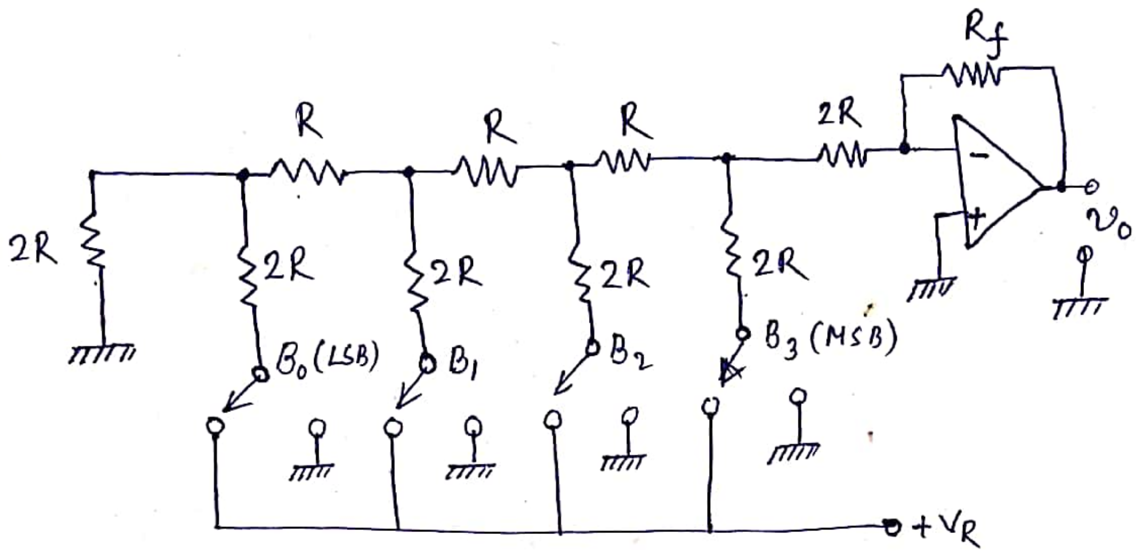
Since G is the virtual ground point, the output voltage is

$$v_0 = -iR_f = -\frac{V_R R_f}{R}\left(2^3 B_3 + 2^2 B_2 + 2^1 B_1 + 2^0 B_0\right) \quad \text{--(2)}$$

Thus the output voltage is proportional to the numerical value of the binary input.

X.

D/A Converter using R-2R Ladder :-



The output voltage is given by

$$v_o = -\left(\frac{R_f}{3R}\right)\cdot\frac{V_R}{2^4}\left(2^3 B_3 + 2^2 B_2 + 2^1 B_1 + 2^0 B_0\right).$$

B_3	B_2	B_1	B_0
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0

B_3	B_2	B_1	B_0
1	1	0	1
1	1	1	0
1	1	1	1

$$\therefore V_0 = -\frac{R_f V_R}{48R}\left(2^3 B_3 + 2^2 B_2 + 2^1 B_1 + 2^0 B_0\right)$$