Unit of Intensity: Bel and decibel :-

Intensity of sound is the flow of sound energy per sec through unit area kept perpendicular to the direction of flow of sound wave.

Bel and Decibel: The bel and decibel are logarithmic units of relative intensity. If the ratio of intensities of two sounds be 10:1 the difference in intensities is said to be 1 bel. If I_1 and I_2 be the intensities of two sounds then the number of bels corresponding to I_1/I_2 is given by

$$N = \log_{10} I_1/I_2$$

A decibel (dB) is one tenth of a bel. The no. of decibel corresponding to I_1/I_2 is given by

$$dB = 10 \log_{10} \frac{I_1}{I_2}$$

Intensity level: Relative intensity is usually expressed in units of intensity level. The intensity level of a sound is the ratio of its intensity to the standard intensity I_0. The value of I_0 for sound waves in air is 10^{-12} watt/ $m^2 = 10^{-16}$ watt/cm^2. This value of I_0 called the threshold value, corresponds to the lower limit of intensity for audibility

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in air at a frequency of 1000 Hz.

The logarithm of the ratio (I/I_0) to the base 10 denotes the intensity level in bel. So

$$\text{Intensity level in bel} = \log_{10}(I/I_0)$$

and $$\text{Intensity level in dB} = 10\log_{10}(I/I_0)$$

Loudness of Sound: Phon:

Intensity of sound is purely physical quantity whereas loudness is different. Loudness of sound depends on its intensity. In general loudness increases with intensity.

Phon is the unit of loudness. For measuring the loudness of a sound in phon, another pure tone of frequency 1000 Hz is taken. The intensity level of a pure tone be n decibel, the loudness level of the original sound is supposed to be n phon.

Fourier's Theorem:

Fourier's Theorem may be stated as: Any single valued periodic function which is continuous or has a finite number of finite discontinuities, may be expressed as a summation of simple harmonic functions having frequencies which are multiples of the frequency of the given function.

Analytically, the Theorem may be written as

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}\left(a_n \cos n\omega t + b_n \sin n\omega t\right) \quad --(1)$$

where the Constants a_n and b_n are to be determined from Euler's formulas and $\omega = \frac{2\pi}{T}$, T being the time period.

The infinite series (1) is called Fourier Series.

The Constants are given by

$$a_0 = \frac{2}{T}\int_0^T f(t)\,dt \quad -----(2)$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\,dt \quad ---(3) \quad (n = 0, 1, 2 \cdots)$$

and $$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\,dt \quad ---(4) \quad (n = 1, 2, 3 \cdots)$$

Square Wave:

The form of square wave is defined by

$$f(t) = \frac{1}{2}a \quad \text{for } 0 < t < T/2$$

$$= -\frac{1}{2}a \quad \text{for } T/2 < t < T$$

(4)

In This case $f(t)$ has a finite discontinuity at $t=0$, $T/2$ and T. The square wave is shown in Fig-1.

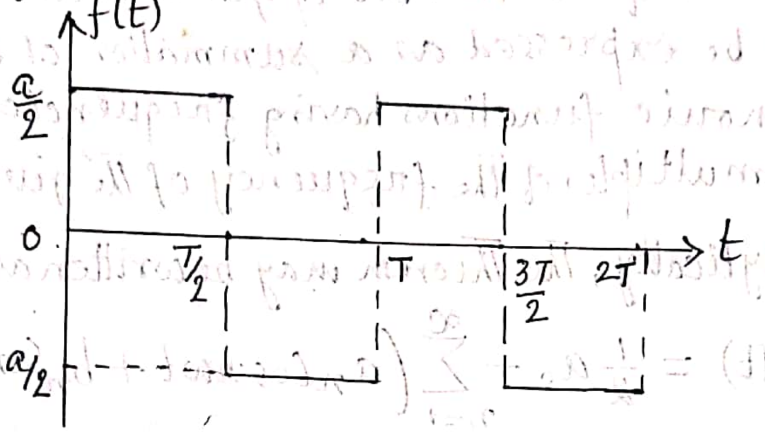

Fig-1.

Let us evaluate the Fourier coefficients a_0, a_n and b_n with the help of the equations (2) (3) and (4).

From eq$\underline{n}$ (2)

$$a_0 = \frac{2}{T}\int_0^T f(t)\,dt = \frac{2}{T}\left[\int_0^{T/2}\frac{a}{2}\,dt + \int_{T/2}^{T}\left(-a/2\right)dt\right]$$

$$= \frac{2}{T}\left[\frac{a}{2}\cdot\cancel{\frac{T}{2}} - \frac{a}{2}\cancel{\frac{T}{2}}\right] = 0$$

From eq$\underline{n}$ (3)

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\,dt$$

$$= \frac{2}{T}\left[\int_0^{T/2}\frac{a}{2}\cos n\omega t\,dt + \int_{T/2}^{T}\left(-a/2\right)\cos n\omega t\,dt\right]$$

$= 0$ for all n

From eq$\underline{n}$ (4)

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\,dt$$

$$\therefore b_n = \frac{2}{T}\left[\int_0^{T/2} \frac{a}{2}\sin n\omega t\,dt + \int_{T/2}^{T}\left(-\frac{a}{2}\right)\sin n\omega t\,dt\right]$$

$$= -\frac{a}{n\omega T}\left[2\cos n\pi - 2\right] \quad \text{Since } \omega T = 2\pi$$

$$= 0 \text{ when } n \text{ even}$$

$$= \frac{2a}{n\pi} \text{ when } n \text{ odd.}$$

Thus the Fourier Series of the square wave is

$$f(t) = \frac{2a}{\pi}\left[\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \cdots\right]$$

It is be noted that the given function $f(t)$ is a discontinuous one but its Fourier series is a superposition of the functions all of which are continuous. The given function $f(t)$ has a finite discontinuity at $t=0$, $T/2$ and T. But the series has definite values equal to zero at these points. It is also to be noted that this value zero is the mean value of the function at the discontinuity.

Saw-tooth wave :-

The 'Saw-tooth' wave is shown in Fig-2

The function $f(t)$ increases linearly from 0 to a in time T and then sharply falls to zero and the whole is repeated again and again.

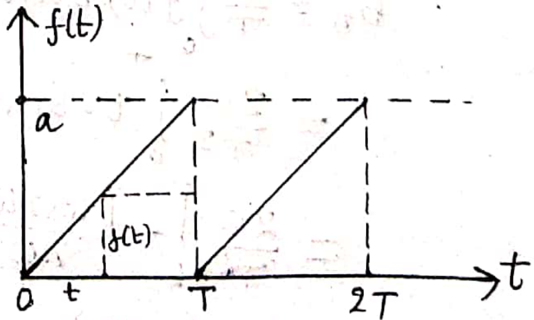

Fig-2

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In This case

$$\frac{f(t)}{t} = \frac{a}{T} \Rightarrow f(t) = \frac{a}{T}\cdot t \text{ from } t=0 \text{ to } T$$

The function $f(t)$ repeats itself with the period T.
The Fourier expansion is

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t$$

$$\therefore a_0 = \frac{2}{T}\int_0^T f(t)dt = \frac{2}{T}\int_0^T \frac{a}{T}t\, dt = \frac{2a}{T^2}\cdot\frac{T^2}{2} = a$$

$$\therefore \boxed{a_0 = a}$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\, dt$$

$$= \frac{2}{T}\int_0^T \frac{a}{T}t\cdot\cos n\omega t\, dt$$

$$= \frac{2a}{T^2}\int_0^T t\cdot\cos n\omega t\, dt$$

$$= \frac{2a}{T^2}\left[\frac{t}{n\omega}\sin n\omega t\right]_0^T - \frac{2a}{T^2 n\omega}\int_0^T \sin n\omega t\, dt$$

(Integrating by parts)

$$= \frac{2a}{T^2}\left[\frac{t}{n\omega}\sin n\omega t + \frac{1}{n^2\omega^2}\cos n\omega t\right]_0^T$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\, dt$$

$$= \frac{2a}{T^2}\int_0^T t\cdot\sin n\omega t\, dt$$

$$= \frac{2a}{T^2}\left[-\frac{t\cos n\omega t}{n\omega}\right]_0^T + \frac{2a}{T^2}\int_0^T \frac{\cos n\omega t}{n\omega}dt$$

(Since $\omega T = 2\pi$)

$$= \frac{2a}{T^2}\left(-\frac{T}{n\omega}\right) + 0 = -\frac{2a}{\omega T n} = -\frac{2a}{2\pi n} = -\frac{a}{\pi n}$$

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$$\therefore \boxed{b_n = -\frac{a}{\pi n}}$$

Therefore,

$$f(t) = \frac{1}{2}a - \frac{a}{\pi}\left\{\sin\omega t + \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t + \cdots\right\}$$

It is to be noted that the value of the Fourier series at the discontinuity, that is at $t=0$ or $t=T$ is the mean of the values of function on two sides of the discontinuity.