Electricity and Magnetism

Unit 8: Electro-magnetic Induction

Ohm's Law:- To make a current flow we have to push on the charges. In response to a push, how fast they move depends on the nature of the material. For most substances the current density $\vec{J}$ is proportional to the force per unit charge $\vec{f}$. $\vec{J} = \sigma \vec{f}$ .... (1)

Here, σ is the proportionality constant that varies from one material to another, it is called the conductivity of the medium. The reciprocal of σ is called the resistivity of the medium. $\rho = 1/\sigma$.

The force that drives the charges to produce the current is usually an electromagnetic force. So, equ(1) can be written as,

$$\vec{J} = \sigma (\vec{E} + \vec{v} \times \vec{B})$$

The velocity of the charges is usually small and we can ignore the second term.

$$\therefore \vec{J} = \sigma \vec{E} \quad .... (2)$$

This is known as the Ohm's law.

If a wire is isotropic and homogeneous, the electric field $\vec{E}$ inside it will be longitudinal and will have same value at all points along the wire. Thus we can write the potential difference between the ends of the wire as, $V = El$. Where l is the length of the wire with the cross-sectional area A. Here the current density will also be uniform. So the current flowing in the wire is,

$$I = JA = \sigma EA = \frac{\sigma A}{l} V = \frac{A}{l\rho} V = V/R \quad \ldots (3)$$

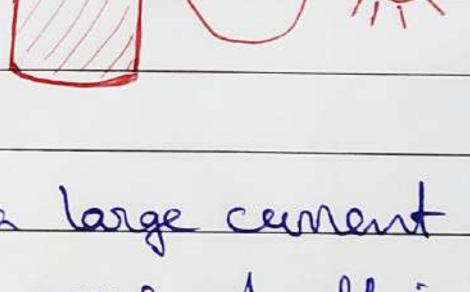
Where $R = \rho l/A$ is the resistance of the wire. This illustrate that the current flowing through the wire is proportional to the potential difference between the ends of the wire.

$$V = IR \quad \ldots (4)$$

This is the more familiar version of Ohm's law.

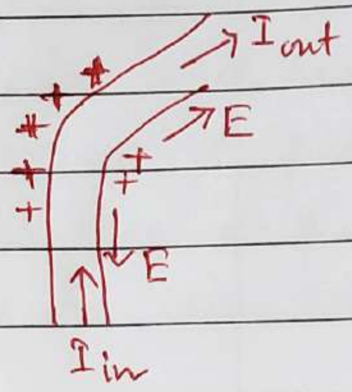
Electromotive Force (emf):

We consider a typical electric circuit with a battery connected to a light bulb. At any moment the current is same all the way around the loop. Here the only driving force is inside the battery.



One might expect that only a large current is produced in the battery and none at all in the lamp. Then who is pushing it in the rest of the circuit and why this push is exactly right to produce the same current in each segment? If the current is not same all around the loop, then charge will pile up somewhere, and the electric field of this accumulating charge will try to make the flow uniform.

Suppose that the current into the bend is greater than the current out. Then charge piles up at the knee, and this produces a field aiming away from the kink. This field will oppose the current flowing in and will promote the current flowing out. So, the field will increase I_{out} and will decrease I_{in}, until these currents become equal, and at this point there will be no further accumulation of charges. It is a system, automatically make the current uniform, and it does it so quickly that we can safely assume the current is same everywhere of the circuit.



The outcome of all this is that there are two forces involved in driving the current around a circuit — the source, $\vec{f_s}$, which is generally confined to one part of the loop (a battery), and the electrostatic force, which smooth out the flow and communicate the influence of the source to distant parts of the circuit.

$$\therefore \vec{f} = \vec{f}_s + \vec{E}$$

The physical agency responsible for $\vec{f}_s$ can be any one of many different things — in a battery it is a chemical force, in a thermocouple it is a temperature gradient, in a photoelectric cell it is light, etc. Whatever the mechanism, its net effect is determined by the line integral of $\vec{f}$ around the circuit.

$$\mathcal{E} = \oint \vec{f} \cdot d\bar{l} = \oint \vec{f}_s \cdot d\bar{l}$$

(Since $\oint \bar{E} \cdot d\bar{l} = 0$ for electrostatic fields, it does not matter whether we use $\vec{f}$ or $\vec{f}_s$). Here $\mathcal{E}$ is called the electromotive force or emf. of the circuit. $\mathcal{E}$ is actually not a force — it is the integral of a force per unit charge. Within an ideal source of emf (a resistanceless battery, say) the net force on the charge is zero (here the conductivity $\sigma = \infty$), so $\vec{E} = -\vec{f}_s$. The potential difference between the terminals 'a' and 'b' of the source is therefore,

$$V = -\int_a^b \vec{E} \cdot d\vec{l} = \int_a^b \vec{f}_s \cdot d\vec{l} = \oint \vec{f}_s \cdot d\vec{l} = \mathcal{E}.$$

We can extend the integral to the entire loop because $\vec{f_s}=0$ outside the source. The function of a battery is to establish and maintain a voltage difference equal to the electromotive force. The resulting electrostatic field drives the current around the rest of the circuit.

Faraday's Law of electromagnetic Induction—

Faraday observed experimentally that whenever the magnetic flux linked with an electric circuit changes, it causes a current induced in the circuit. This phenomenon is known as the electromagnetic induction. The experimental results led to the development of two laws.

1. The induced emf in a circuit is ~~[illegible]~~ proportional to the rate of change of magnetic flux linked with the circuit.
2. The direction of the induced emf is such that it will oppose the change of flux producing it.

The second law is known as Lenz's law which dictates the direction of Induced emf.

If ϕ is the magnetic flux linked with the circuit at time t, then the laws of electromagnetic induction can be expressed mathematically as

$$\varepsilon = -d\phi/dt,$$

where ε is the induced emf. The negative sign signifies that the emf, ε, opposes the change of flux. Thus the induced emf is often referred as the back emf.

If C is a closed circuit binding an open surface S, and placed in a magnetic field $\vec{B}$. The flux ϕ through S is given by

$$\phi = \int_S \vec{B}\cdot d\vec{s} \quad \cdots (1)$$

The induced emf generated around C is

$$\varepsilon = -d\phi/dt \quad ----(2)$$

If $\vec{E}$ is the electric field in space, then by definition the induced emf around C can be written as

$$\varepsilon = \oint_C \vec{E}\cdot d\vec{l} \quad ---(3)$$

Here $d\bar{l}$ is an elemental length in C.

From eq(2) and (3) we have

$$-\frac{d\phi}{dt} = \oint_c \bar{E}\cdot d\bar{l}$$

$$\text{or, } -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{s} = \oint_c \vec{E}\cdot d\vec{l} \qquad [eq(1)]$$

$$\therefore \oint_C \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{s} \quad \cdots (4)$$

This is the integral form of Faraday's law of electromagnetic induction. If the circuit is fixed then the time derivative can be moved inside the integral. Here the time derivative becomes a partial derivative since $\vec{B}$ is a function of position aswell.

$$\therefore \oint_C \vec{E}\cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s} \quad \ldots (5)$$

Now, using Stokes theorem we can write,

$$\int_S (\vec{\nabla}\times\vec{E})\cdot d\vec{s} = -\int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s} \quad \ldots (6)$$

This is true for any arbitrary fixed surface S.

$$\therefore (\vec{\nabla}\times\vec{E}) = -\frac{\partial \vec{B}}{\partial t} \quad \ldots (7)$$

This is differential form of Faraday's law of electromagnetic induction.