Why can an electron not exist inside the nucleus?

The mathematical form of the Heisenberg's Uncertainty Principle can be written as,

$$\Delta x \Delta p_x \geq \frac{h}{2\pi}, \quad \dots\dots (1)$$

which can be stated as, "The product of the uncertainty Δx in the position of a particle at a certain instant and the uncertainty Δp_x in its momentum at the same instant is equal to or greater than $h/2\pi$. h is Planck's constant, $h = 6.626 \times 10^{-34}\ m^2 kg/s$

We know that the radius of the nucleus is approximately 10^{-14} m. If an electron is to exist inside the nucleus, then the uncertainty in the position of the electron is,

$$\Delta x = 10^{-14}\ m.$$

According to the uncertainty principle

$$\Delta x \Delta p_x = h/2\pi$$

$$\text{or, } \Delta p_x = \frac{h}{2\pi\Delta x} = \frac{6.626 \times 10^{-34}}{2 \times 3.14 \times 10^{-14}}\ kg.m/s$$

$$\text{or, } \Delta p_x = 1.06 \times 10^{-20}\ kg.m/s.$$

So, the uncertainty in the momentum of electron should be of the order of 10^{-20} kg.m/s.

An electron with this much high momentum must have a velocity comparable to the velocity of light. So, the energy of the electron should be calculated by the following formula.

$$E = \sqrt{m_0^2c^4 + p^2c^2} = \sqrt{(9.11 \times 10^{-31})^2(3 \times 10^8)^4 + (1.06 \times 10^{-20})^2(3 \times 10^8)^2}$$

$$= \sqrt{(6722.4 \times 10^{-30}) + (10.11 \times 10^{-24})}$$

$$\simeq \sqrt{10.11 \times 10^{-24}}$$

$$= 3.18 \times 10^{-12}\ J$$

$$= \frac{3.18 \times 10^{-12}}{1.6 \times 10^{-19}}\ eV$$

$$\therefore E = 19.88\ MeV \simeq 20\ MeV$$

If the electron exists in the nucleus, it should have an energy about 20 MeV. However, it is observed that electrons (beta-particles) ejected from the nucleus during β-decay have energies of approximately 3 MeV. Also, experimental results show that no particle in the atom can have energy greater than 4 MeV. Therefore, it is impossible for an electron to exist inside the nucleus.

Nuclear Forces

According to Coulomb's law the closely spaced positively charged protons within the nucleus should repel each other strongly and they should fly apart. Since stable nucleus exists, it is therefore difficult to explain it unless one assumes that the nucleons are under influence of some very strong attractive forces acting between them. The forces inside the nucleus, binding neutrons to neutrons, protons to protons and neutrons to protons, are classified as strong interactions and are represented as n-n, p-p and n-p. Experimental evidence by Japanese scientist Hideki Yokawa indicated that these forces are essentially equal in magnitude.

According to Yukawa, the nuclear forces have the following characteristics:-

1. The nuclear forces are short range, i.e, effective only at short ranges.
2. They are charge independent, i.e, they do not seem to depend on the charge of the particle.
3. Nuclear forces are the strongest forces known in nature.
4. They are readily saturated by the surrounding nucleons.
5. They are spin-dependent.

We will now discuss the above characteristics in more details.

Short Range -!

The result of the p-p, n-p scattering experiments show that the nuclear forces operate over very short distances. Between two nucleons the distance is $\sim 10^{-15}$ m or less. If a nucleus is bombarded with protons, then the protons that do not pass too close to the nucleus are scattered by electric repulsive forces. But if the energy of the incident protons are high enough to overcome Coulomb repulsion, they may pass very close to the nucleus, and fall in the range of attractive nuclear forces. They would then be captured and fall into the potential well of the nucleus. The scattering of protons in this case is mainly due to strong and attractive nuclear forces.

There is however some evidence that at extremely short distances ($\sim 0.5\times10^{-15}$ m) the attractive force turns into a repulsion, so that in a stable nucleus the nucleons do not get too close together.

Charge independence -)

Experimental evidence indicates that the interaction between two nucleons is independent of the charge. The nuclear forces acting between two protons, or between two neutrons or between a neutron and a proton are same to a high degree of accuracy.

Strong forces :-

The gravitational and electromagnetic interaction were known to us long before the nuclear forces, as they were associated with macroscopic bodies. But they are far weaker compared to the nuclear force. The force between the nucleons are the strongest force found in nature. It is about 10^{40} times stronger than the gravitational force.

Saturation :-

Each nucleon within a nucleus interacts with a limited number of nucleons nearest to it. The ability of nuclear forces to act upon other particles saturate when the nucleon gets completely surrounded by other nucleons. The nucleons, that are located outside the surrounding nucleons do not feel the interaction of the surrounded nucleon. Nuclear forces are the only forces in nature that show a saturation effect.

Spin-dependent :-

The nuclear forces depend on the mutual orientation of spins of various nucleons and are different in parallel and antiparallel spins.

According to Yukawa's theory, the protons and neutrons do not exist independently within the nucleus. They constantly exchange charges by emission and absorption of π-mesons (pions). This results in an exchange of virtual mesons by nucleons, within the nucleus, in ultra short time intervals ($\sim 10^{-23}$ to 10^{-24} s). As the exchange occurs in a very short time, the uncertainty principle requires that no visible change in nucleonic mass would be observed. This gives rise to rapid meson exchange or meson field between protons and neutrons in which meson acts as a quantum of nuclear force. This process is analogous to exchange of photons between charged particles in electromagnetic interactions.

Liquid Drop Model :-

The liquid drop model of the nucleus was proposed by Niels Bohr and Fritz Kalcker in 1937. This model was later applied by C.F. von Weizsäcker and H.A. Bethe to develop a semi-empirical formula for the binding energy of the nucleus. The macroscopic properties of the nucleus, such as the density of the nuclear matter and the constant binding energy per nucleon are very similar to those found in a liquid drop. The similarities between the nucleus of an atom and a liquid drop are the following:

1. The density of nuclear matter is independent of the size of the nucleus, similar to the density of liquid which is independent of the volume of the drop.
2. The attractive force near the surface of the nucleus is similar to the surface tension of the liquid drop.
3. The constant binding energy per nucleon is analogous to the latent heat of vaporization of liquid.
4. The nucleus is supposed to be spherical in shape in the stable state, similar to a liquid drop.
5. The formation of a compound nucleus by the absorption of a bombarding particle is similar to the process of condensation from vapor to liquid phase in case of liquid drop.
6. Due to the radioactive properties of the nucleus different particles are emitted from it. This is similar to the emission of molecules from the liquid drop during evaporation.
7. The energy of the nuclei is analogous to the internal thermal vibrations of the molecules of the liquid drop.
8. The nucleons inside the nucleus interacts only with the neighbouring nucleons similar to the molecules inside a liquid drop.