$$f(x) = a_0 + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \cdots$$
$$+ b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + \cdots$$

We find the average value on $(-\pi, \pi)$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{a}{2\pi}\int_{-\pi}^{\pi} dx + a_1 \cdot \frac{1}{2\pi}\int_{-\pi}^{\pi} \cos x\,dx$$
$$+ \cdots + b_1 \frac{1}{2\pi}\int_{-\pi}^{\pi} \sin x\,dx.$$

All integrals on the right hand side are zero except the first.

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = a_0 \frac{1}{2\pi}\int_{-\pi}^{\pi} dx$$
$$= a_0.$$

$$\therefore \quad a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx.$$

$$A_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx.$$

$$a_0 = \frac{A_0}{2}$$

$$f(x) = \frac{A_0}{2} + a_1 \cos x + a_2 \cos 2x + \cdots$$
$$+ b_1 \sin x + b_2 \sin 2x + \cdots$$

Multiply both sides by $\cos x$ and find the average value of each term.

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\cos x\,dx = a_0 \cdot \frac{1}{2\pi}\int_{-\pi}^{\pi}\cos x\,dx$$

$$+ a_1 \cdot \frac{1}{2\pi}\int_{-\pi}^{\pi}\cos^2 x\,dx + a_2\frac{1}{2\pi}\int_{-\pi}^{\pi}\cos 2x\cos x\,dx$$

$$+ b_1\frac{1}{2\pi}\int_{-\pi}^{\pi}\sin x\cos x\,dx.$$

Only this term exists.

$$\therefore \quad a_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos x\,dx.$$

$$\boxed{a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx}$$

$n = 0 \Rightarrow$ ?

H.W $\quad b_n = ?$ $\quad \boxed{b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx}$

H.W $\quad f(x) = \begin{cases} 0, & -\pi < x < 0 \\ 1, & 0 < x < \pi. \end{cases}$