Testing for Bivariate Normal

Suppose two variables x and y from B.N.D

$(\mu_1, \mu_2; \sigma_1^2, \sigma_2^2, \rho)$

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma_1\sigma_2} e^{-\frac{1}{2(1-\rho^2)}\left\{\left(\frac{x_i-\mu_1}{\sigma_1}\right)^2 - 2\rho\left(\frac{x_i-\mu_1}{\sigma_1}\right)\left(\frac{y_i-\mu_2}{\sigma_2}\right) + \left(\frac{y_i-\mu_2}{\sigma_2}\right)^2\right\}}$$

$\mu_1, \mu_2 \in R$, $0 < \sigma_1, \sigma_2 < \infty$, $-1 < \rho < 1$

- Here conside R.S from $BN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$
- of size n (x_1, y_1) (x_2, y_2) . . (x_n, y_n)

Here we want to test difference for μ_x and μ_y if x and y variables are measure in same unit

consider a test $H_0: \mu_x - \mu_y = \xi_0$ ag.

$H_1: \mu_x - \mu_y \neq \xi_0$

Here Let $Z = X - Y$, $\sim N(\mu_x - \mu_y, \sigma_y^2 + \sigma_x^2 - 2\rho\sigma_x\sigma_y)$

Here $z_i = x_i - y_i$, $\bar{z} = \bar{x} - \bar{y}$

$$S_z^2 = \frac{1}{n-1}\sum(z_i - \bar{z})^2.$$

Here our problem is reduces to test $H_0: \mu_z = \xi_0$ ag $\mu_z \neq \xi_0$

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Where σ_z unknown

So test statistic is t

$$T = \frac{\sqrt{n}(\bar{z} - \xi_0)}{s_z} \sim t_{n-1}$$

The test procedure is for $H_0: \mu_z = \xi_0$ ag. $\mu_z \neq \xi_0$ is same as earlier ($H_0: \mu = \xi_0$ ag. $H_1: \mu \neq \xi_0$)

i.e, if $|T| > t_{\frac{\alpha}{2}, n-1}$ then reject H_0 at level α ag. $H_1: \mu_z \neq \xi_0$

$$w = \{ : |T| < -t_{\frac{\alpha}{2}, n-1} \text{ or } |T| > t_{\frac{\alpha}{2}, n-1} \}$$

for $H_0: \mu = \xi_0$ ag. $H_1: \mu > \xi_0$

$$w = \{ T > t_{\alpha, n-1} \}$$

$$= \{ z: \frac{\sqrt{n}(\bar{z} - \xi_0)}{s} > t_{\alpha, n-1} \}$$

for $H_0: \mu = \xi_0$ ag. $H_1: \mu < \xi_0$

$$w = \{ z: \frac{\sqrt{n}(\bar{z} - \xi_0)}{s} < -t_{\alpha, n-1} \}$$

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This is known as paired t test.

Real life example

Suppose weights of n patients before they are subjected to change a diet and after a lapse of six month are given. Then to test any significant change in weight game we can use paired t-test.

- Testing for $\rho = 0$

Here $r = \dfrac{\frac{1}{n}\Sigma x_i y_i - \bar{x}\bar{y}}{\sqrt{V(x)}\sqrt{V(y)}}$ sample correlation coefficient

For $H_0: \rho = 0$ ag. $H_1: \rho \neq 0$

Under $H_0: \rho = 0$; $\dfrac{r\sqrt{n-2}}{\sqrt{1-r^2}} \overset{H_0}{\sim} t_{n-2}$

So $t = \dfrac{r\sqrt{n-2}}{\sqrt{1-r^2}}$ $\overset{H_0}{\sim} t_{n-2}$ is ~~test~~ statistic. under H_0 $E(t) = 0$

i.e under H_0: we expect

$t \simeq 0$

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So. Here if $|t| > t_{\frac{\alpha}{2}; n-1}$ we

(as earlier process) we reject

$H_0: \rho = 0$ ag. $H_1: \rho \neq 0$ at level α.

So critical region is

$$\omega = \left\{ (x,y) : \left| \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \right| > t_{\frac{\alpha}{2}; n-1} \right\}$$

• Suppose (X_1, X_2) has a Bivariate Normal distribution with unknown parameter $(\mu_1, \mu_2, \sigma_1, \sigma_2, \rho)$ test $H_0: \beta = 0$ ag. $H_1: \beta \neq 0$ where β is the regression coefficient of X_2 on X_1

Soln

$$E(X_2 \mid X_1 = x) = \mu_2 + \rho \frac{\sigma_2}{\sigma_1}(x - \mu_1)$$

is the regression equation of X_2 on X_1

$$\beta = \rho \frac{\sigma_2}{\sigma_1}$$

$H_0: \beta = 0 \Rightarrow \rho \frac{\sigma_2}{\sigma_1} = 0$ i.e $\rho = 0$

(as $\frac{\sigma_2}{\sigma_1} > 0$)

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so testing is reduces to $H_0: \rho = 0$ vs. $\rho \neq 0$

• Testing for $\frac{\sigma_x}{\sigma_y}$

$\{(X_1, Y_1), (X_2, Y_2) \ldots (X_n, Y_n)\}$ be the R.S of size n from $BN(\mu_1, \mu_2, \sigma_1, \sigma_2, \rho)$

Here $H_0: \frac{\sigma_x}{\sigma_y} = \xi_0$ vs. $H_1: \frac{\sigma_x}{\sigma_y} \neq 0$

Let $U = X + \xi Y$

$V = X - \xi Y$

where $\xi = \frac{\sigma_x}{\sigma_y}$.

Here U and V are jointly distributed as normal variables

with $cov(U, V) = \sigma_x^2 - \xi^2 \sigma_y^2 = 0$

So under H_0: $\frac{\sigma_x}{\sigma_y} = \xi_0$

$U = X + \xi_0 Y$

$V = X - \xi_0 Y$

Here $cov(U, V) = 0$ i.e. $\frac{\sigma_x^2}{\sigma_y^2} = \xi_0^2$

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Soln. $H_0: \frac{\sigma_x}{\sigma_y} = \xi_0$ ag. $H_1: \frac{\sigma_x}{\sigma_y} \neq \xi_0$

reduces to $H_0: \rho_{uv} = 0$

Here test statistic is

$$t = \frac{r_{uv}\sqrt{n-2}}{\sqrt{1-r_{uv}^2}}$$

So the test is as earlier [See earlier test pro]

(Ex) Suppose the weights of n pigs before and after application of a special food suppliment in a farm are recorded. under the suitable assumption provide a test procedure for testing whether the variability in the pig weight before and after applying food supplement remain unchanged

[Hint:- use test procedure for $H_0: \sigma_x = \sigma_y$ i.e $H_0: \frac{\sigma_x}{\sigma_y} = 1$

ag $H_1: \frac{\sigma_x}{\sigma_y} \neq 1$.]

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